CATION SELECTIVE GLASS ELECTRODES AND THEIR MODE OF OPERATION

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I. INTRODUCTION

The present paper attempts a three fold task. First, it summarizes the most important general properties of cation responsive glass electrodes in the steady state at room temperature. Second, it proposes an atomic mechanism by which such electrodes operate. Third, it provides information of a practical nature in conjunction with a bibliography to recent applications. This material represents one aspect of a continuing study of the elementary atomic origin of equilibrium ionic specificity. The conclusions regarding cation responsive glass electrodes are based upon work which is to be published in detail elsewhere (Eisenman, 1961a). The theoretical analysis of the general factors controlling ionic specificity calls upon principles enunciated initially in collaboration with J. U. Casby and D. O. Rudin (Eisenman, Rudin and Casby, 1957c; Rudin and Eisenman, 1959).

It is hoped that this paper will stimulate interest not only in the practical application of cation responsive glass electrodes but also in the consideration of such glass electrodes and analogous physico-chemical systems as models for the understanding of cationic effects in biology. I believe that to the extent that the present analysis proves useful in understanding the origin of the potential and ion exchange properties of glasses and minerals, it is of relevance to biologists who wish to understand the origin of their phenomenological counterparts of cellular ionic accumulation and membrane potentials. For in both the living cell and in the glass the energetically important nearest neighbors to cations are predominantly oxygens and it is a relatively trivial difference that these oxygens are held on a framework of C and P in living cells as opposed to the Al and Si in the usual glasses and minerals. It is therefore my opinion that the development of *theoretical methods* for understanding the mode of operation of glass electrodes constitutes as important an "Emerging Technique in Biophysics" as does the more obvious use of cation responsive glass electrodes as biophysical measuring devices.

History

Hydrogen ion responsive glass electrodes, the essential components of which are

silica and an alkali metal oxide have been used extensively since their discovery by M. Cremer (1906) and characterization by Haber and Klemensiewicz (1909) three years later. A well established feature of these electrodes is the presence of a slight sensitivity to the alkali metal cations which manifests itself as an "alkaline error" at high pH (Hughes (1922), Horovitz (1923), Schiller (1924), MacInnis and Dole (1930), Sokolov and Pasynskii (1932)). In the pursuit of glasses having a minimal alkaline error a number of workers noticed the deleterious effect of small amounts of Al_2O_3 or B_2O_3. Thus, Horowitz and Schiller showed that the introduction of B_2O_3 caused glass electrodes to become nearly as sensitive to Na^+ as to H^+; and Hughes (1928) explicitly noted that the less Al_2O_3 the glass contained the better it was for measuring H^+. These observations were extended by Lengyel and Blum (1934) who, through a careful and systematic study of the chemical compositions of simple glasses, conclusively demonstrated that the introduction of Al_2O_3 or B_2O_3 (or both) into the glass caused its potential to become strongly dependent upon Na^+ as well as H^+ but concluded incorrectly that further variation of glass composition beyond introducing 10% Al_2O_3 or B_2O_3 had no important effect. They also reported that in such electrodes K^+ and Li^+ had a variable influence which they concluded was less reproducible and characteristic than that of Na^+. Contemporaneously with Lengyel and Blum, Evstrop'ev and Suikovskaja (1934) according to Shul'ts (1953) also demonstrated that the addition of B_2O_3 resulted in cation responsiveness. The pioneering studies of Lengyel and Blum were extended by other authors who were primarily interested in the existence of Na^+ sensitivity *per se* (i.e., the Na^+ selectivity relative to H^+ only) and in establishing whether or not the electrodes were reversible in the thermodynamic sense. An excellent review of this work is given by Shul'ts (1953), whose studies and those of Nicolskii and Tolmacheva (1937a, b, c) are particularly noteworthy. In fact, Shul'ts convincingly demonstrated by direct comparison with a sodium amalgam electrode, that typical cation responsive glasses at neutral and alkaline pH behave as reversible electrodes for Na^+.

Less attention has been given to the *relative sensitivity* (sometimes called "specificity" or "selectivity") of the glasses among the various alkali metal cations and it was not until 1957 that it was recognized that a systematic relationship existed for the selectivities among the various group Ia cations as a function of glass composition (Eisenman, Rudin, and Casby 1957a, 1957b). This realization served to characterize various glass compositions, some of which had sufficiently high selectivities for Na^+ or K^+ to be of practical use for the measurement in ionic mixtures of Na^+ activity on the one hand or K^+ activity on the other.

2. GENERAL CHARACTERISTICS OF CATION RESPONSIVE GLASS ELECTRODES

The Equation for the Electrode Potential and Definition of Relative Sensitivity

The relative sensitivity of glass electrodes to various ions has been defined in terms

of the K_n parameter of equation 1, which was developed as an empirical expression capable of describing the potential in mixtures of any two univalent cations for most glass electrodes and also in certain three ion mixtures in which one ion is held at constant activity, e.g., Na^+–K^+ mixtures at constant pH (Eisenman, et al. 1957a).

$$E = E_c + \frac{RT}{F} \ln [(A^+)^{1/n} + K_n^{1/n}(B^+)^{1/n}]^n \tag{1}$$

Parentheses around the ionic species A^+, B^+ denote the activities of the ions in aqueous solution. A particular value of K_n (and also of n) characterizes the potential of any glass composition for a given pair of ions.[1] Equation 1 has also been derived theoretically by the present author by generalizing somewhat the original treatment of Nicolskii (1937). The derivation is presented in the appendix, which also presents the derivation of a formally equivalent equation for ion exchange which has been found to describe the equilibria of many ion exchangers (cf. Equation 2 of Eisenman, et al. 1957a). Some properties of equation 1 are presented in Figure 1, and representative glass electrode data are given in Figure 3. The upper portion of Figure 1 indicates the effect of varying the product $K_n(B^+)$ for a constant value of $n = 1$, while the lower portion indicates the effect of varying n for a constant value of the product $K_n(B^+)$. Equation (1) is completely symmetrical between (A^+) and (B^+) and predicts a theoretical slope of RT/F as a function of ln (A^+) or ln (B^+) whenever the solution is "purely" (A^+) or (B^+), (i.e., when $(A^+) \gg K_n(B^+)$ or $K_n(B^+) \gg (A^+)$). Of course, K_n and the limits of accuracy desired determine what constitutes a negligible amount of B^+ in the presence of A^+ and *vice versa*. For chemically stable glasses, equation 1 is generally valid for Li^+, Na^+, K^+, Ag^+ and NH_4^+ at least over the pH range 1–11.[2] However, in the case of Rb^+ and Cs^+, deviations set in at extremes of pH as discussed elsewhere (Eisenman, 1961a).

Equation (1) indicates that the *activities* of the cations govern the potential. This was found to be true in the range of concentrations (below 1.0 molar) studied previously (Eisenman, et al. 1957a). The validity of this statement has now been extended to concentrated solutions by measurements in pure solutions of LiCl, NaCl, KCl, NaBr, and KBr with representative glass electrode compositions referred to the appropriate silver-silver halide half cell. Typical data obtained from such concentration cells without liquid junction are presented in Figure 2 for the mean activity coefficients. The experimental data determined with the glass (points) can be seen to agree sufficiently well with the solid curves extracted from the compilation of Robinson and Stokes (1959) to establish that it is the activity of the cation to which the electrodes respond throughout the entire concentration range, even though the

[1] The terminology has been modified slightly from the original in that the subscript n has been added to the constant of relative sensitivity, K_n; and the subscripts A and B have been omitted (although they will be written explicitly whenever needed for clarity). Thus, the present K_n (or K_{nAB}) corresponds to the original K_{AB}; and n (or n_{AB}) corresponds to the original n_{AB}.

[2] One must, of course, calculate any association of such ions as Ag^+ and NH_4^+ which occurs at high pH.

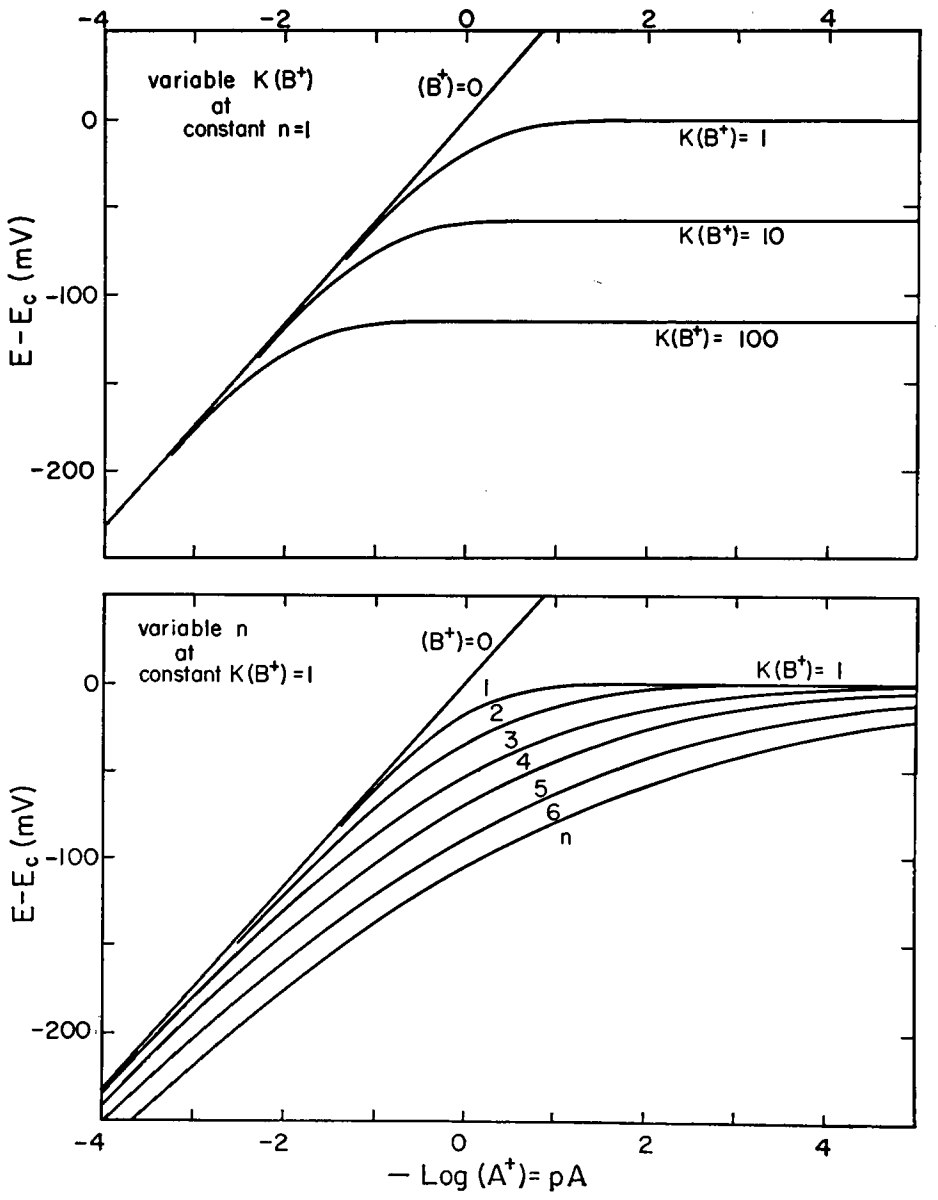

FIGURE 1 Some properties of Equation 1. The potential in two-ion mixtures is plotted on the ordinate as a function of the Logarithm of the activity of ion A^+ for various constant products of (B^+) and K_n (subscript "*n*" not labeled in the figure) and values of *n*. The products can represent different concentrations of B^+ for constant K_n values or *vice versa*. Note that when $(B^+) = O$ (or when $K_n = O$) the potential depends linearly on $-\text{Log}\ (A^+)$. The converse is also true since Equation 1 is completely symmetrical with respect to (A^+) and (B^+).

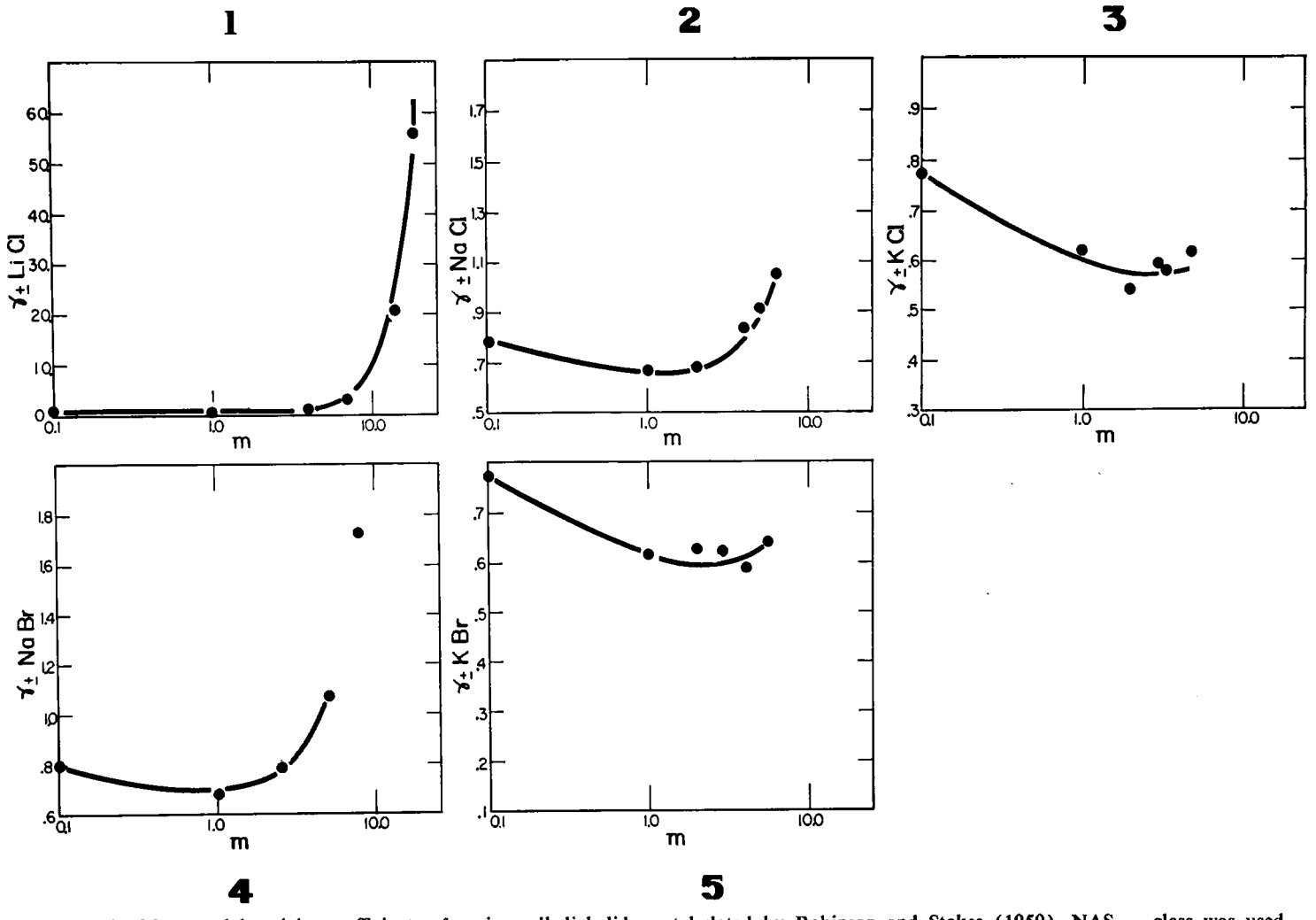

FIGURE 2 Mean molal activity coefficients of various alkali halides as measured at 25° ± 0.5° C at neutral pH by cation responsive glass electrodes in conjunction with an Ag; Ag Halide reference electrode (points) in comparison with the solid curves drawn from the data tabulated by Robinson and Stokes (1959). NAS_{11-18} glass was used in Li^+ and Na^+ solutions (subfigures 1, 2, 4); NAS_{27-4} (Beckman 78137) glass was used in K^+ solutions (subfigures 3, 5).

observed mean activity coefficients encompass values as small as 0.6 for KCl and larger than 50 for LiCl. (The measurements presented in Figure 2 were made with dip type electrodes at 25° ± 0.5°C and are being repeated more precisely with flow-through electrodes of Portnoy-Angst design (Portnoy, Thomas, and Gurdjian (1961a)) at 25° ± 0.1°C in an oil thermostat.)

Characterization of Group Ia *and* H^+ *Selectivity of* Na_2O—Al_2O_3—SiO_2 *Glasses*

In an aqueous solution, H^+ is always present. Consequently, studies of potential in "pure solutions" of the individual alkali metal cations are actually performed in mixtures of two ions. Often, the presence of H^+ is sufficiently negligible that the mixture may be thought of as being a "single cation" solution, but in many cases the H^+ has an important effect. For this reason, it is necessary in characterizing electrodes to measure the response of the electrode to the various cations as a function of H^+ activity. In our previous work the effect of H^+ was shown to be negligible at neutral pH in 0.1 N solutions of Li^+, Na^+ and K^+; and the relationship between composition and cation selectivity for these ions was therefore characterized approximately through the simple comparison of the potentials in 0.1 N solutions of the individual cations at neutral pH and the potential in 0.1 N HCl. The precise values of K_n and also of n have now been determined for the group Ia cations. They are summarized in Figures 4 and 5 by locating the K_n and n value for each electrode at the appropriate composition point and then interpolating smooth contours of iso-K_n or of iso-n values. The determination of K_n and n for each composition point is based upon the measurement of potential of several electrodes of each glass composition as a function of pH for a constant 0.1 N cation concentration (and also constant ionic strength).[3] Representative data are illustrated in Figure 3 in which the points are experimental while the solid curves are drawn according to equation 1. Notice in Figure 3 that in the presence of a constant activity of alkali cation the potential becomes a constant, independent of pH, in the limit to the right as H^+ approaches zero. This limiting potential is characteristic of each cation for a given glass, and the millivolt difference between this potential and that in pure 0.1 N HCl (*i.e.* the potential of the diagonal straight line at pH 1.08) gives K_n directly by equation 2.

$$(2) \qquad E_{H^+=0.1} - E_{\substack{B^+=0.1 \\ \lim H^+ \to 0}} = -\frac{RT}{F} \ln K_{nHB}$$

The selectivities of the various cations relative to H^+ and relative to each other are interrelated through equation (3) (Eisenman, et al. 1957a).

$$(3) \qquad K_{nAB} = \frac{K_{nHB}}{K_{nHA}}$$

[3] The agreement of data from different electrodes of the same composition is remarkably good, usually falling within the ± 10 mV limits of accuracy used in characterizing the general compositional properties of the glasses.

The limiting potentials for the various cations in Figure 3 can be seen to fall into different sequences of selectivity for different glass compositions (designated by the Roman numerals for each composition in Figure 3). Thus, in the high pH limit NAS 11–18 selects among the group Ia cations in the sequence X (Na > Li > K > Rb > Cs), while NAS 27–7.8 responds in the sequence IV (K > Rb > Cs > Na > Li). The significance of these sequences will be discussed later.

Note that Figures 4 and 5 are plotted in *atoms percent* of the cations of the glass structure (as determined by chemical analysis of the final glass) in place of the previous units of *moles percent* of the oxides to make easier the task of theoretical interpretation.[4] Since Figures 4 and 5 constitute a complete characterization of the parameters governing the steady state potential behavior of the Na_2O—Al_2O_3—SiO_2 system, they make it possible to reconstruct the potential behavior of any desired glass in any ionic concentrations.[5]

A Further Systematization of Glass Electrode Selectivity

While the data of Figures 4 and 5 are in themselves systematically dependent upon composition, an important further simplification can be made by examining simultaneously the selectivity among all the cations as a function of glass composition. For example, consider the selectivities encountered as a function of atoms % Al when moving through the composition fields of Figure 4 along the line labelled "50(Si)" for which all glasses have a constant 50 atoms % Si. These selectivities are presented in Figure 6 as a function of the atoms % Al in the glass (which reciprocates with the atoms percent Na, the sum of Na and Al being 50 atoms %). Observe that when Al is zero, as in the usual pH responsive glasses, the electrodes are highly specific for H^+ relative to the other cations ($K_{_{H,B}} \leq 10^{-12}$). But as one increases the Al content, the electrodes rapidly become highly group Ia responsive and the relative

[4] The manner of converting between atoms percent and moles percent is illustrated in the following example. A glass $(Na_2O)_{20}$ $(Al_2O_3)_{10}$ $(SiO_2)_{70}$, whose composition is 20 mole percent Na_2O, 10 Mole percent Al_2O_3, and 70 Mole percent SiO_2 (corresponding to the designation NAS 20–10) has its sodium, aluminum, and silicon atoms present in the ratio 40:20: 70, as determined by multiplying the numbers of atoms in the oxide formula by their subscript. Since the sum of atoms is 130, the atom percent values for Na^+, Al^{+++}, and Si^{++++} are given by the ratios 40/130; 20/130; 70/130 which yield an atom percent composition Na 30.77; Al 15.38; Si 53.85.

[5] A detailed characterization will be presented elsewhere (Eisenman, 1961a) for other singly charged cations such as NH_4^+, Ag^+, Tl^+ to which the present glasses are highly responsive (being in some instances 10^5 times as selective for Ag^+ as for H^+ and responding to Ag^+ in concentrations as low as 10^{-9} M). The response to NH_4^+ follows approximately that to K^+, but ranges from 10 times as selective to NH_4^+ as to K^+ for "low field strength" glasses to 10 times as selective for K^+ as NH_4^+ in "high field strength" compositions. Specific responses have also been observed to various amines and amino acids and these have been found to vary with glass composition in a series of preliminary experiments. The sensitivity to doubly charged ions of the present glasses is relatively slight, but Garrels and his colleagues have recently developed glasses of improved divalent responsiveness. (Garrels, Sato, Thompson and Truesdell, 1961).

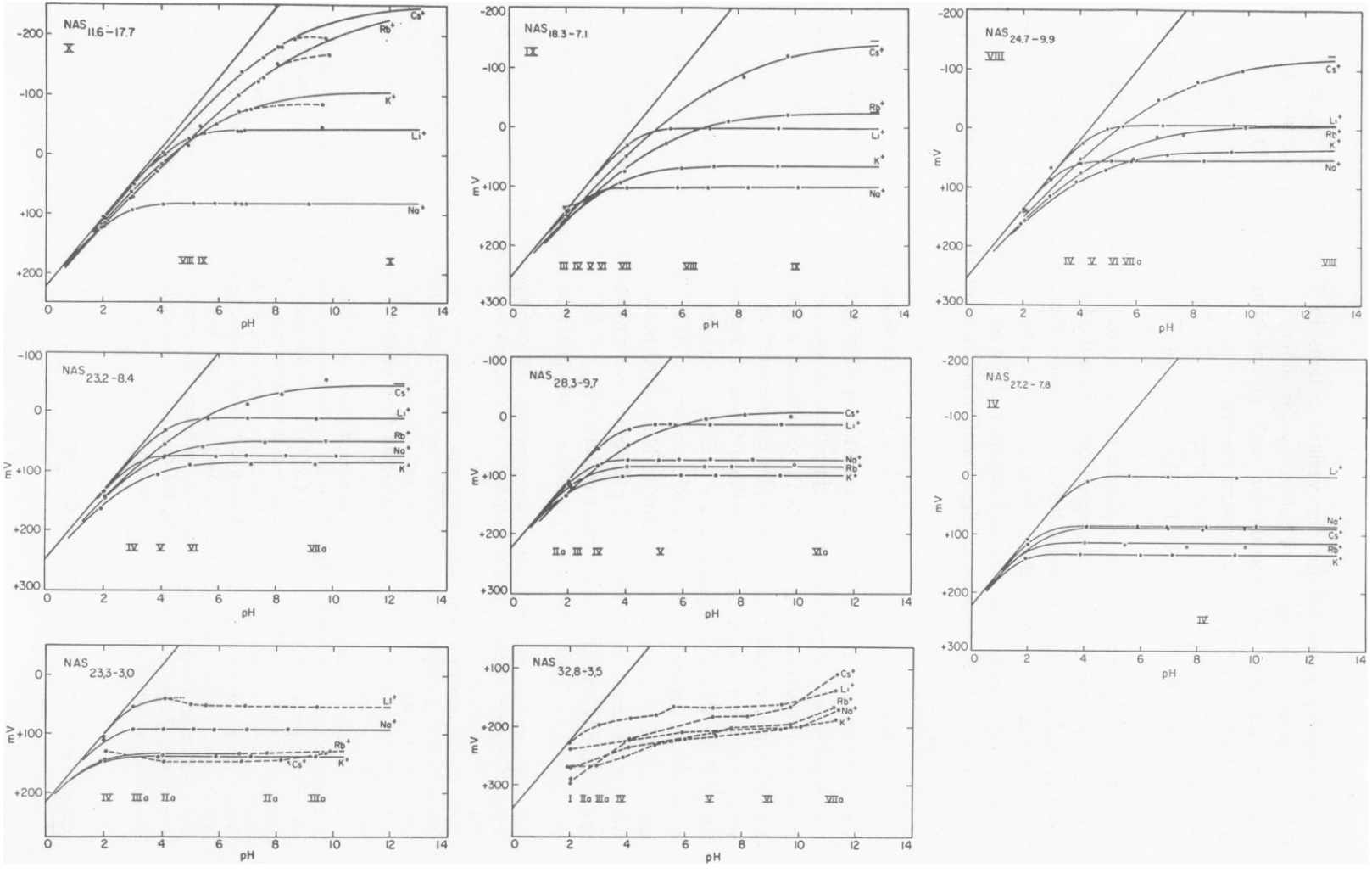
NAS 11.6 – 17.7
X
Cs+
Rb+
K+
Li+
Na+
VIII IX
X
NAS 18.3 – 7.1
IX
Cs+
Rb+
Li+
K+
Na+
III IV V VI VII
VIII
IX
NAS 24.7 – 9.9
VIII
Cs+
Li+
Rb+
K+
Na+
IV V VI VII a
VIII
NAS 23.2 – 8.4
Cs+
Li+
Rb+
Na+
K+
IV V VI
VII a
NAS 28.3 – 9.7
Cs+
Li+
Na+
Rb+
K+
II a III IV
V
VI a
NAS 27.2 – 7.8
IV
Li+
Na+
Cs+
Rb+
K+
IV
NAS 23.3 – 3.0
Li+
Na+
Rb+
Cs+
K+
IV III a II a
II a
III a
NAS 32.8 – 3.5
Cs+
Li+
Rb+
Na+
K+
I II a III a IV
V
VI
VII a
mV
pH

FIGURE 3 Representative Sodium Alumino-silicate glass electrode potential data at 22° ± 2° C as a function of pH in 0.1N solutions of the indicated cations. The points are experimentally measured by the indicated glass electrodes referred to a saturated KCl; Calomel reference electrode (Beckman asbestos fiber type). The pH was measured simultaneously with a variety of commercial glass electrodes. The solid curves are drawn according to Equation 1. Dashed curves indicate data deviating significantly from the predictions of Equation 1. The composition of each glass is designated in the upper left of the subfigures in Mol % of Na_2O and Al_2O_3 respectively (SiO_2 equals the remainder). The Roman numerals below these designations indicate the selectivity rank order of K_n pertaining at the high pH limit. The Roman numerals below the curves indicate the rank orders of K' observed at the indicated pH. (Note the dependence of the selectivity pattern upon pH). The six upper glasses are chemically durable compositions which obey Equation 1 well. The two lower glasses are soluble in H_2O and deviate significantly from Equation 1, although the deviations are least severe for Na^+ and K^+. (Despite the complexities of the data of the soluble glasses, note that the pattern of selectivity rank orders persists).

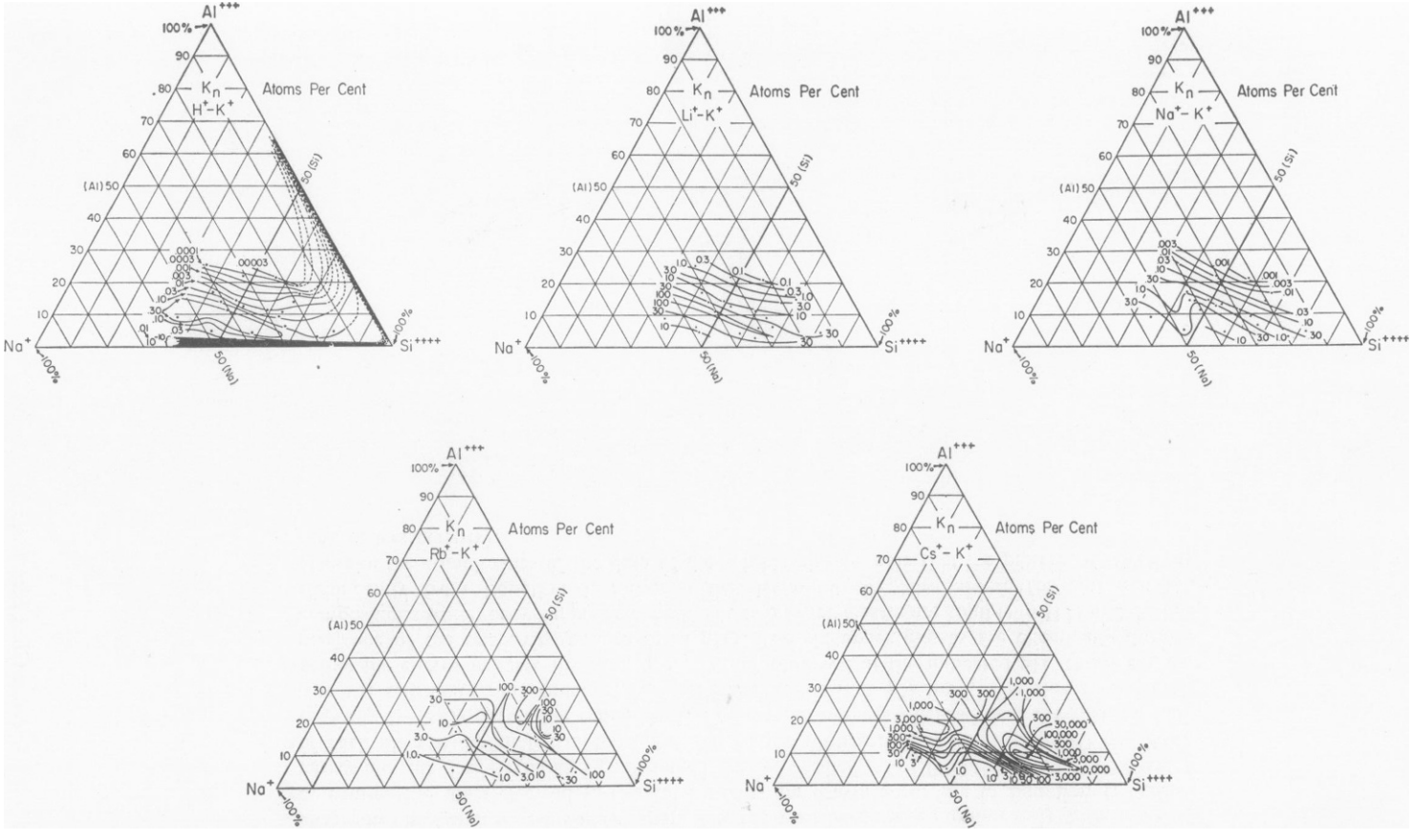

FIGURE 4 Contours of isosensitivity (K_n) at 22° ± 2° C in the steady state in the Na_2O—Al_2O_3—SiO_2 glass composition field. The coordinates are in Atoms % (at 10% intervals) of Na, Al, and Si as described in the Text. The corners of the triangle represent 100% Na_2O, Al_2O_3 or SiO_2. Points represent all those glass compositions (gravimetrically analyzed by U. S. Testing Company from aliquots of the final, fused glass) for which K_n values have been experimentally determined. The solid contour lines of isosensitivity are curvilinear interpolations between the K_n data obtained at these points. (The dashed contour lines in the case of H^+ — K^+ mixtures at the upper left represent a prediction of the glass behavior in the unexplored composition regions). The numerical values of each contour refer to the extent to which the first ion of the pair dominates the potential behavior (i.e. it is $K_{n_{BA}}$ or the reciprocal of $K_{n_{AB}}$ where A^+ is the first written ion and B^+ the second written ion on the subfigure heading).

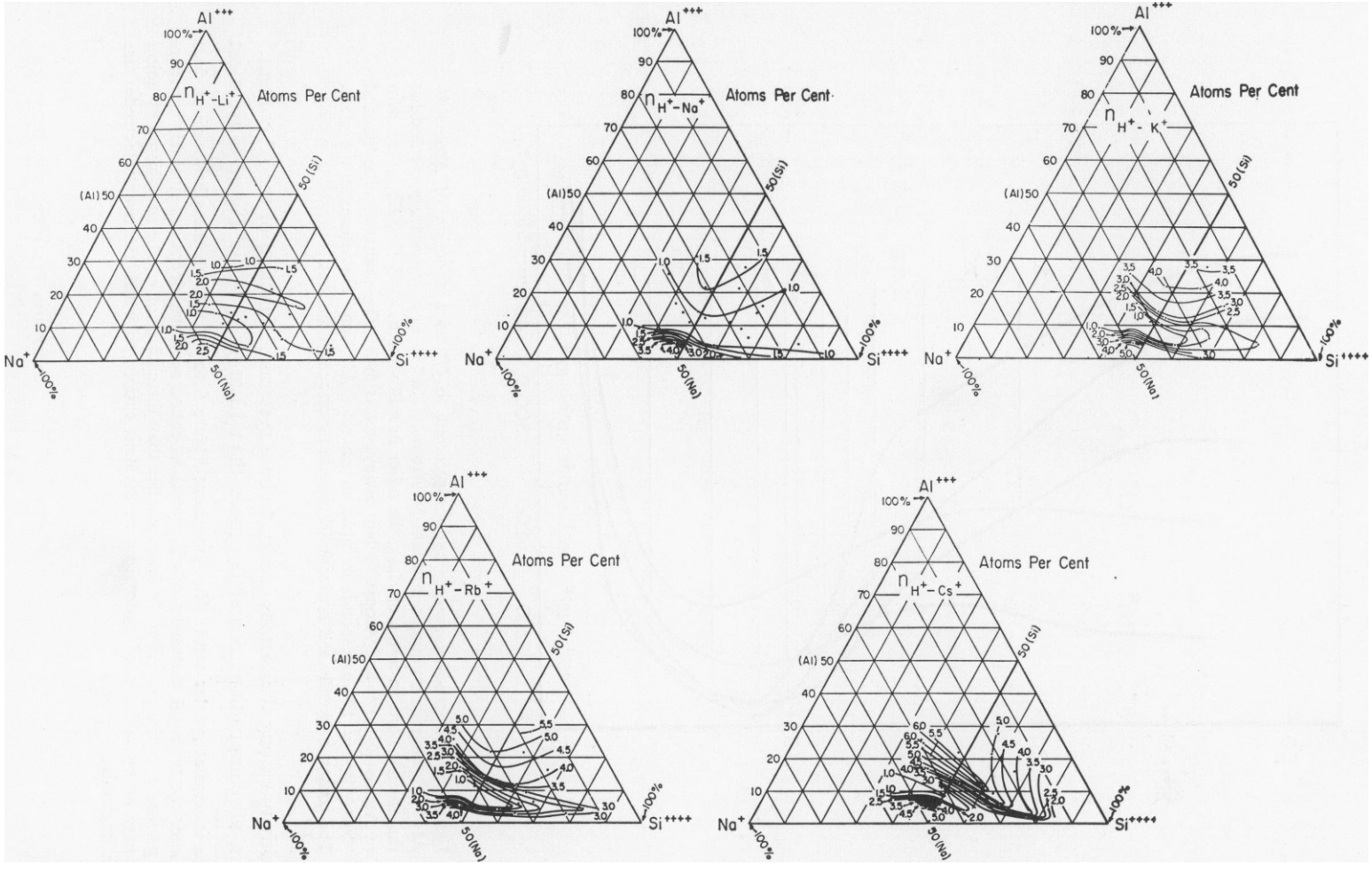

FIGURE 5 Contours of iso-n values at 22° ± 2° C in the steady state in the Na_2O—Al_2O_3—SiO_2 system plotted in the same manner as the data of Fig. 4. However, the numerical designations indicate the value of n in the region adjacent to the contour line, rather than on it. Thus all compositions lying between 1.0 and 1.5 have values of n equal to 1, those between 2.0 and 2.5 have the values of n equal to 2, etc. The n values are accurate to ± 0.5 in most cases.

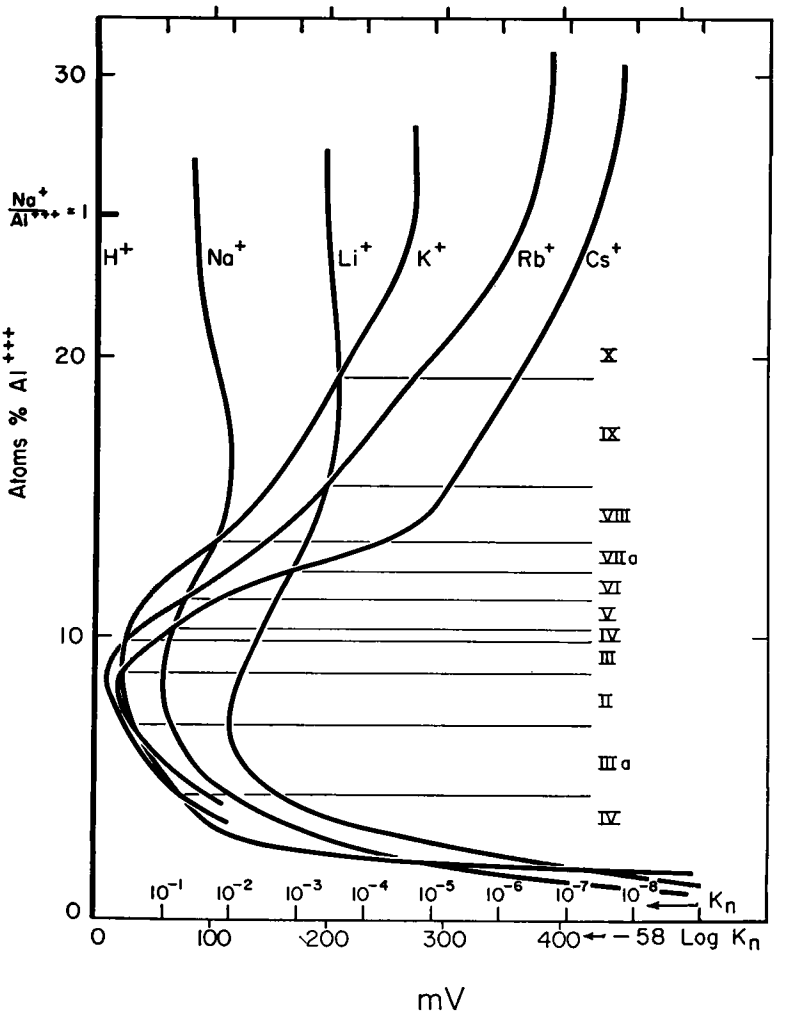

FIGURE 6 Selectivities among cations referred to H^+ as a function of Atoms % Al^{+++} in glasses containing a constant 50 Atoms % Si^{++++}. Two scales are presented for the abscissa, one in terms of $K_{n_{HB}}$, the other in millivolts as -58 Log $K_{n_{HB}}$. The sum of Atoms % Al^{+++} and Atoms % Na^+ always totals 50%. The ordinate value labelled "$Na^+/Al^{+++} = 1$" corresponds to equal numbers of Al^{+++} and Na^+ atoms in the glass. The roman numerals and horizontal lines are described in the text.

sensitivity among the cations is seen to be a systematic function of the Al content.[6] Thus, all compositions containing more than 13 atom % Al in Figure 6 are most

[6] The somewhat greater sensitivity throughout Figure 6 to H^+ than to the alkali cations does not alter the practical usefulness of the glasses as cation sensitive electrodes since H^+ is generally present at a concentration 10^5 times smaller than the cations or can be so adjusted by anion exchange or by addition of hydroxides of divalent cations to which the usual electrodes are quite insensitive.

responsive to Na^+ among the alkali cations. Between 10 and 13 atoms % Al the glasses are chiefly responsive to K^+. While between 4 and 10 atoms % Al they are primarily selective for Rb^+. Examining Figure 6 still more closely, note that each cation isotherm crosses the others sequentially. This leads to composition regions distinguished by having different rank orders of cation selectivity. Such regions are delimited by the horizontal lines drawn to each intersection and labelled by Roman numerals whose significance is discussed below.

Not only can one observe such systematic relationships in Figure 6, but one can in fact describe the entire Na_2O—Al_2O_3—SiO_2 composition field in terms of the above selectivity sequences by locating in Figure 7 the sequence observed for the composition point corresponding to each electrode and again interpolating contours to delimit these regions. Figures 6 and 7 reveal the existence of a remarkable simplicity in the pattern of intra-cation selectivity for the entire Na_2O—Al_2O_3—SiO_2 system. In this pattern only 11 out of a possible 120 sequences for the alkali cations are observed. The sequence of cation relative sensitivities in each numbered region of Figures 6 and 7 corresponds to a sequence in a newly discovered pattern of cation effects which has been found applicable to glass electrodes quite generally (cf. Fig. 9) as well as being widely present in nature (Eisenman, et al. 1957c, Rudin and Eisenman, 1959) and to which we will refer in discussing the mode of operation of the glass electrode. It should be noted that the isotherms separating regions of the same selectivity rank order in Figure 7 bear the same relation to glass composition as those governing the selectivity between one alkali metal cation and another in Figure 4. In either case they are predominantly lines of constant Na^+/Al^{+++} ratio. This indicates that one need only specify the Na^+/Al^{+++} ratio to specify all important selectivity properties of a sodium alumino-silicate glass. In fact, it will be shown more generally in a following section that one need only specify the ratio of M^+/Al^{+++} together with a scaling factor, x, characteristic of M^+ and Al^{+++} to specify the selectivity of the other alkali alumino-silicates. It should also be noted that since Na^+–K^+ selectivity is a simple function of Na^+/Al^{+++} ratio as indicated in Figure 4, a measurement of K_n for Na^+ vs. K^+ usually suffices to characterize Na^+/Al^{+++} and therefore to characterize approximately the selectivity for all other cations as well. This is elaborated upon below.

$K'_{Na,K}$ *as a General Index of Cationic Selectivity*

The above finding that the isotherms of selectivity of Na^+ relative to K^+ depend in the same manner upon glass composition as the contours of the selectivity fields between different selectivity rank orders suggests that the cation selectivity for the glasses might be a simple function of the Na^+ to K^+ selectivity (K^+ and Na^+ are chosen for convenience as the two cations most often studied).[7] This proposition is

[7] This is supported by theoretical considerations discussed in section 3.0 which indicate that the Na^+ to K^+ selectivity should also be an index of the anionic field strength, upon which latter quantity ionic selectivity should depend.

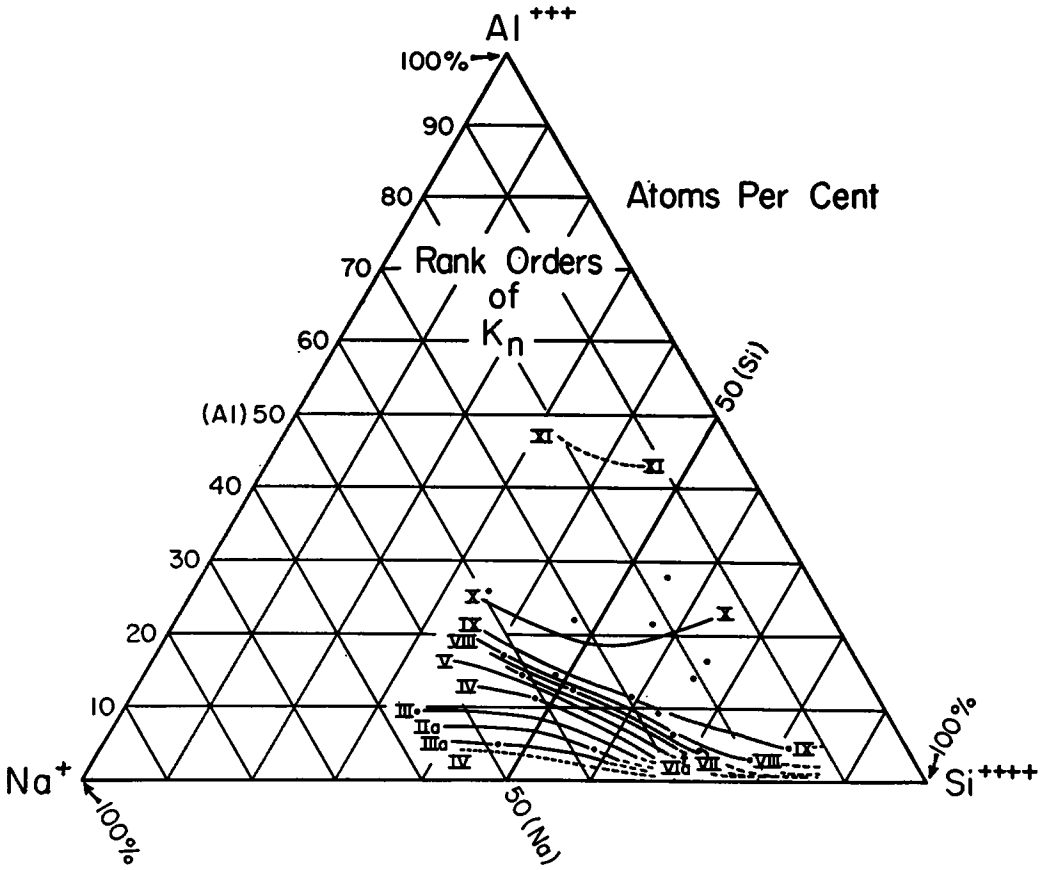

FIGURE 7 Rank order of cationic selectivity as a function of glass composition. The Na_2O—Al_2O_3—SiO_2 system is characterized in the manner of Fig. 4 according to selectivity rank order (e.g. all glasses between the IX and X contours have a cation selectivity in rank order IX, etc.).

tested in Figure 8a for data from electrodes of the Na_2O—Al_2O_3—SiO_2 system whose K_n values for the various cations at neutral pH have been plotted as a function of their selectivity between Na^+ and K^+.[8]

Each vertical set of points in Figure 8a corresponds to the observed selectivities of a given glass electrode whose position along the abscissa is determined by its

[8] The selectivity between Na^+ and K^+ at neutral pH (K'_{NaK}, pH 7) is nearly the same as the limiting selectivity (K_{aNaK}). K'_{NaK} is characterized by the separation between Na^+ and K^+ isotherms observed at pH 7 in Figure 3, and is, of course, the difference of potential between 0.1 N solutions of these cations at neutral pH. Neutral pH was chosen for Figure 8a because this is the pH under which the ion exchange data of the minerals of Figure 8b were obtained.

selectivity between Na^+ and K^+.[9] Visually determined average lines have been drawn through the data points. Notice the dependence upon $K'_{Na,K}$ not only of the sequences of selectivity (indicated by Roman Numerals) but also the quantitative dependence of the selectivity magnitude of each ion. There is, of course, some scatter of the data (particularly noticeable for H^+) resulting from superposing upon one chart all data lying along a given isotherm of Na^+ to K^+ selectivity of Figure 4 (cf. the greater complexity of H^+-K^+ than Na^+-K^+ isotherms there).

The findings of Figure 8a are of practical importance because they restrict to easily manageable proportions the amount of data needed to ascertain the general selectivity properties to be expected of a given glass composition. They are also of theoretical interest as should become clear later. They mean, quite simply, that it is only necessary to measure the Na^+ to K^+ selectivity in order to know the selectivities among all the other cations for most glasses.

Of importance also is the finding that the above systematization is not restricted to the potential properties of the Na_2O—Al_2O_3—SiO_2 system alone, but is also manifested in the ion exchange selectivity of various natural minerals and synthetic inorganic zeolites (which, incidentally, are alumino-silicates) as can be seen in Figure 8b, which presents on the same coordinates as 8a ion exchange selectivity data extracted from the literature. (Barrer, et al. 1950, 1953, 1956; Jenny, 1927, 1932; Rothmund and Kornfeld, 1918; Schachtschabel, 1940; Krishnamoorthy, et al. 1950). The data in Figure 8b represent the free energies corresponding to the experimental "apparent" equilibrium (or distribution) coefficients at neutral pH when the exchangers are 50% exchanged.[10] To emphasize the fact that the same quantitative pattern of selectivity governs mineral exchangers and glass electrode potentials, the isotherms of Figure 8b are directly traced from the potential data of Figure 8a. The extent to which these isotherms satisfactorily represent the ion exchange data is apparent. This means that a measurement of the Na^+ to K^+ selectivity in the ion exchange of a mineral, combined with a knowledge of the selectivity pattern of Figure 8a for sodium alumino-silicate glass electrodes suffices to predict the ion exchange selectivities for the other cations as well. This has obvious implications in designing ion exchangers, as well as setting limits on the selectivities to be expected in them. The above finding, while perhaps startling at first, is actually not unexpected when one recognizes that, at the atomic level, the sites of ion exchange in the alumino-silicate

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[9] Because a maximum exists in the Na^+ to K^+ selectivity, it has been necessary in locating data points to the right of the maximum of the Na^+ isotherm, to use the K^+—Cs^+ selectivity instead of the Na^+—K^+ selectivity. This maximum is postulated in section 3.0 to result from water swelling by the glasses.

[10] The ion-exchange of these minerals has been found to follow equation 2 of Eisenman, et al. 1957a, and therefore the value of the distribution coefficient at 50% exchange is the same as the value of K_n.

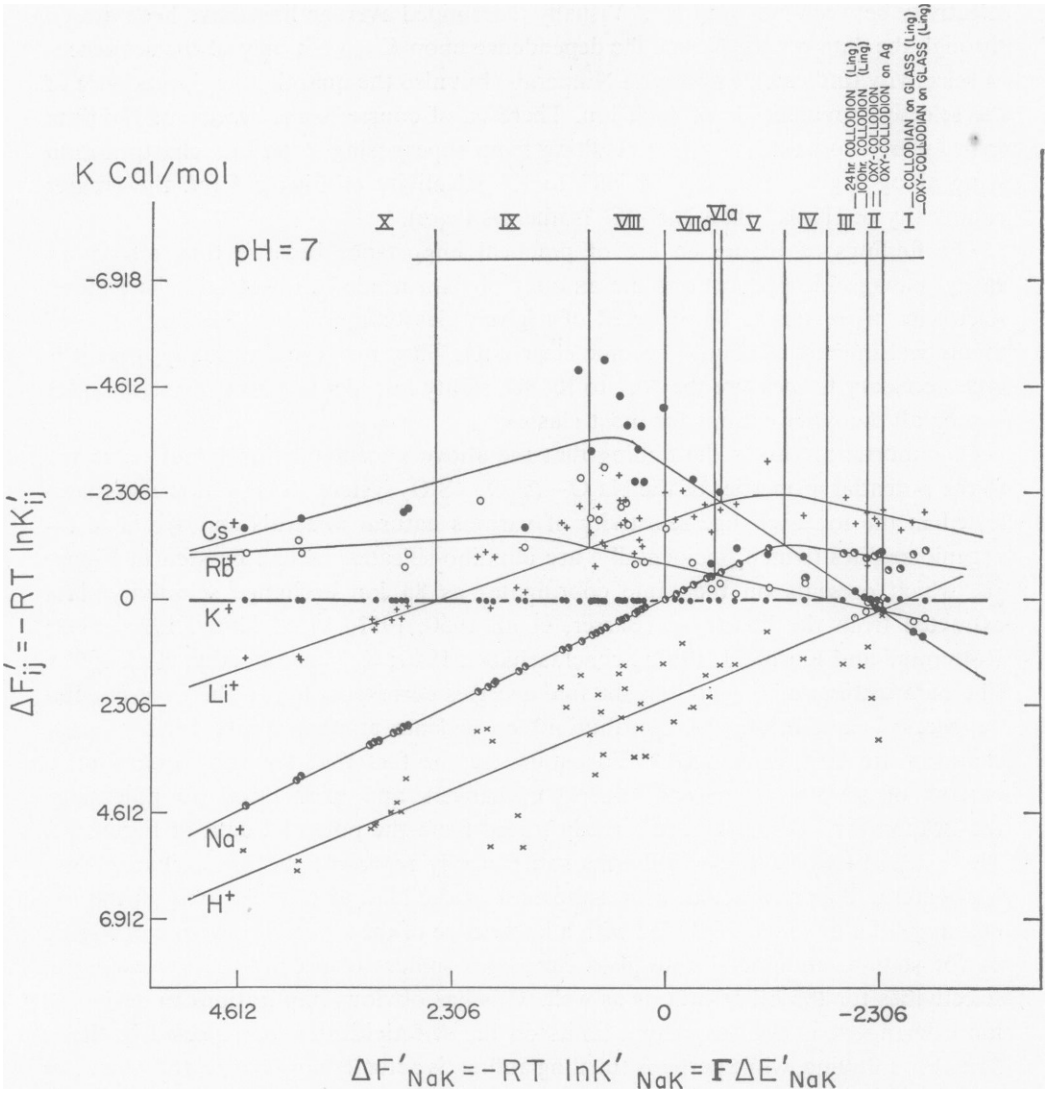

FIGURE 8 *A*. Potential selectivities of Sodium Aluminosilicate glass electrodes and of air-dried collodion electrodes at neutral pH ranked empirically as a function of Na^+ to K^+ selectivity except for those electrodes to the right of the maximum in the Na^+ isotherm, for which the K^+ to Cs^+ selectivity was used, as discussed in the Text. The data were measured as the potential differences between 0.1N solutions of the various cations at pH 7 ($\Delta E'_{NaK}$) which are related to the free energy scales used here by: $\Delta E'_{NaK} = \Delta F'_{NaK}/F$ and to the apparent equilibrium constant at pH 7 by: $\log_{10} K'_{NaK} = -\Delta F'_{NaK}/1.364$. F has the value 2.306×10^4 cal. volt^{-1} equiv.$^{-1}$. Each unlabelled vertical row of data points corresponds to a particular glass electrode; while the data labelled "collodion" represent the potentials of air-dried collodion films after Ling and Kushnir (1961) plated on Ag where specified or on glass where unspecified. (The data labelled "Ling" were kindly made available by Ling and Kushnir). The solid curves are visually drawn averages to the data and the vertical lines correspond to the intersections of these isotherms. Between these intersections the particular selectivity rank orders denoted by the Roman numerals pertain.

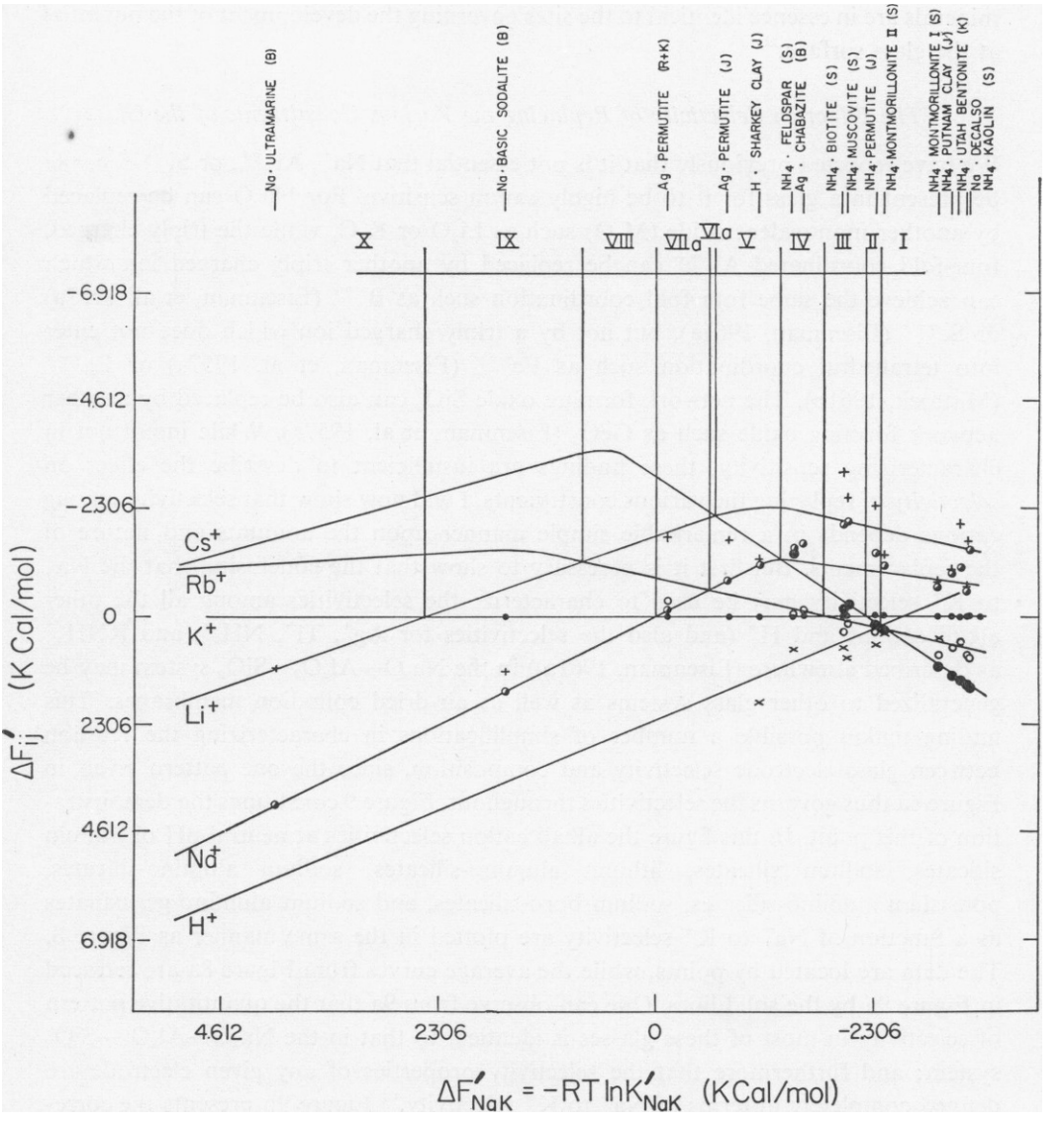

FIGURE 8 *B.* Ion-exchange selectivities of inorganic ion-exchangers at neutral pH, plotted in the same manner as *A* for direct comparison. The solid curves are traced from *A* and the experimental values of K'_{AB} for various cations at 50% exchange were extracted from the literature and located as data points along the abscissa by their $K'_{Na,K}$ value (except for Na-Ultramarine, which was located by K'_{LiNa} and for the six exchangers in rank order I and II for which K'_{KCs} was used). B refers to data of Barrer, et al. (1950, 1953, 1956), R and K to data of Rothmund and Kornfeld (1918), J to data of Jenny (1927, 1932), S to data of Schachtschabel (1940), and K to data of Krishnamoorthy, et al. (1950).

minerals are in essence identical to the sites governing the development of the potential at the glass surface.[11]

The Effect on Selectivity of Replacing the Various Constituents of the Glass

We have reported previously that it is not essential that Na^+, Al^{+++}, or Si^{++++} *per se* be present in a glass for it to be highly cation sensitive. For Na_2O can be replaced by another monovalent oxide (M_2O) such as Li_2O or K_2O, while the triply charged, four-fold coordinated Al^{+++} can be replaced by another triply charged ion which can achieve the same four-fold coordination such as B^{+++} (Eisenman, et al. 1957a) or Sc^{+++} (Eisenman, 1961a), but not by a triply charged ion which does not enter into tetrahedral coordination such as Fe^{+++} (Eisenman, et al. 1957a) or La^{+++} (Mattock, 1961b). The network forming oxide SiO_2 can also be replaced by another network forming oxide such as GeO_2 (Eisenman, et al. 1957a). While important in characterizing sensitivity, these findings are insufficient to describe the effect on *selectivity* of replacing the various constituents. I will now show that selectivity among cations depends in a remarkable simple manner upon the amounts and nature of the replacements. But first it is necessary to show that the conclusion that the Na^+ to K^+ selectivity may be used to characterize the selectivities among all the other alkali cations and H^+ (and also the selectivities for Ag^+, Tl^+, NH_4^+, and RNH_3^+ as described elsewhere (Eisenman, 1961a)) in the Na_2O—Al_2O_3—SiO_2 system may be generalized to other glass systems as well as air-dried collodion membranes. This finding makes possible a number of simplifications in characterizing the relation between glass electrode selectivity and composition, since the one pattern given in Figure 8a thus governs the selectivities throughout. Figure 9 constitutes the demonstration of this point. In this figure the alkali cation selectivities at neutral pH of lithium silicates, sodium silicates, lithium alumino-silicates, sodium alumino-silicates, potassium alumino-silicates, sodium boro-silicates, and sodium alumino-germanates as a function of Na^+ to K^+ selectivity are plotted in the same manner as Figure 8. The data are located by points, while the average curves from Figure 8a are retraced in Figure 9a by the solid lines. One can observe from 9a that the quantitative pattern of selectivity in most of these glasses is identical to that in the Na_2O—Al_2O_3—SiO_2 system; and furthermore that the selectivity properties of any given electrode are defined completely in terms of Na^+ to K^+ selectivity.[12] Figure 9b presents the corresponding (K_n) selectivities in the high pH limit and is of particular interest for the

[11] This statement does not mean to imply that the potential across such a mineral membrane would be identical to that across the corresponding glass, because the mineral is complicated by the presence of aqueous interstitial diffusion channels (whose presence, in fact, permit the minerals to function as reasonably rapid ion exchangers). In a very real sense the internal ion exchange sites of the mineral correspond to an infolded surface of the glass.

[12] This is most true for the Al containing glasses. The boro-silicates appear to manifest a somewhat lower *magnitude* of their selectivities which I believe to be related to greater water swelling (M. Nordberg, personal communication) as discussed in Section 3.

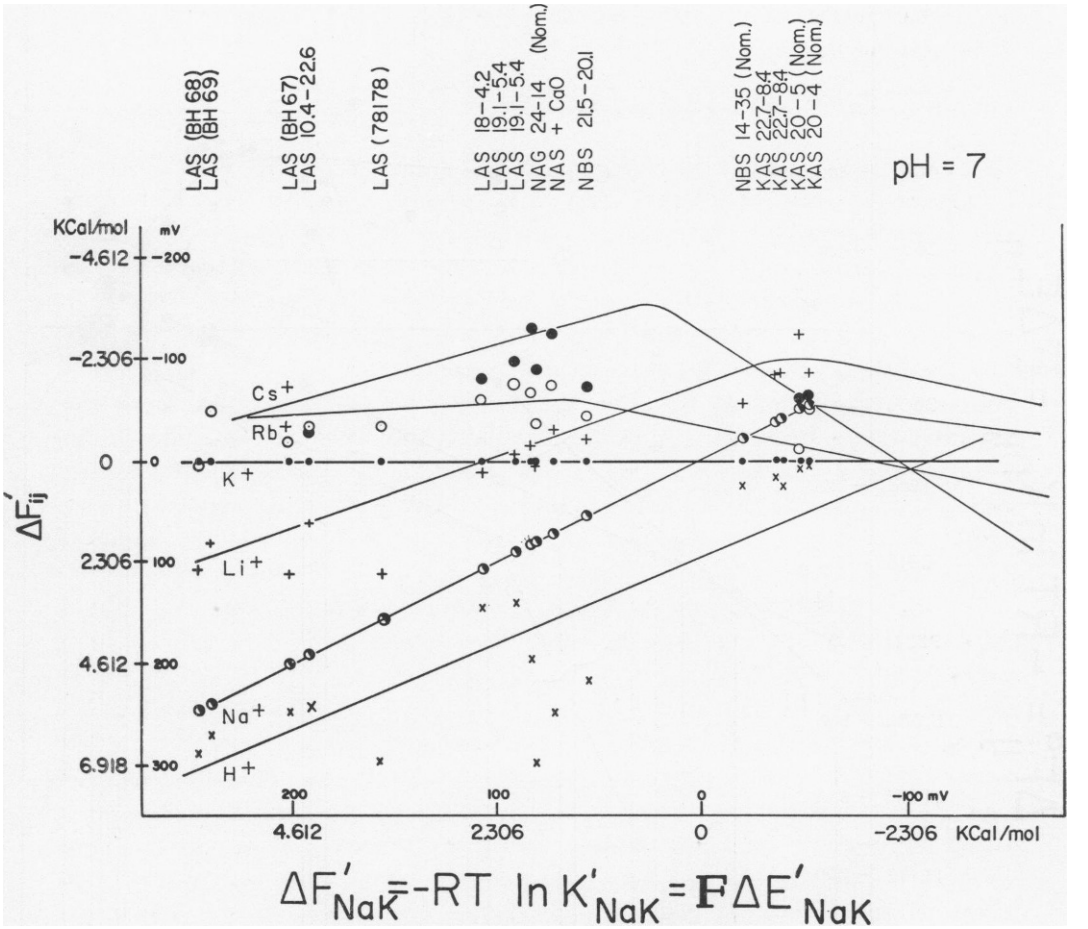

FIGURE 9 *A*. Selectivities of glasses other than Sodium Alumino-silicates at neutral pH superposed upon the average curves for Sodium Alumino-silicates of Fig. 8. The subsidiary scales present the equivalent potential differences in millivolts. Each vertical row of data points corresponds to the designated type of glass. "LAS" stands for Li_2O—Al_2O_3—SiO_2 glasses; "KAS" for K_2O—Al_2O_3—SiO_2 glasses; "NAG" for Na_2O—Al_2O_3—Ge_2O glasses; "NBS" for Na_2O—B_2O_3—SiO_2 glasses; "NAS + CaO" for Na_2O—Al_2O_3—SiO_2—CaO glasses. Where known, the analyzed mole % compositions are indicated. "Nom" indicates that the composition is the nominal, or batch, composition rather than the final, chemically analyzed composition.

typical pH glasses to the left, whose selectivities at neutral pH are obscured by their responsiveness to H^+.

Not only does the selectivity pattern of Figures 8 and 9 hold for the polymerized oxyanion sites of inorganic glasses and minerals but it also applies to the carboxylic sites of air-dried collodion films. For it can be seen in Figure 8a that the group Ia

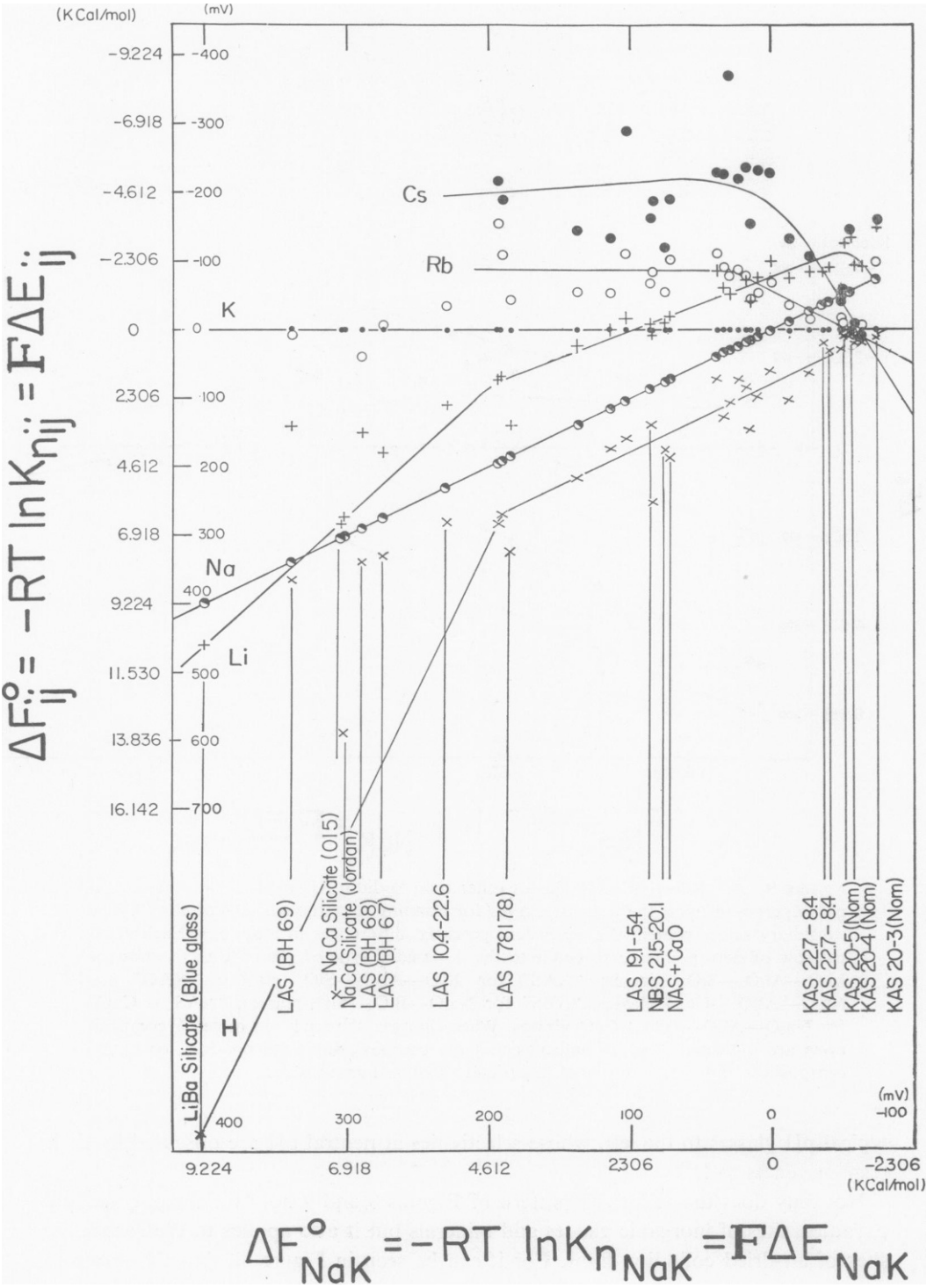

FIGURE 9 *B.* K_n selectivities for representative glasses of various compositions in the high pH limit plotted in the manner of Fig. 8. Unlabelled glasses are Sodium Alumino-silicates. Others are labelled as in *A*, but note the additional three typical pH electrodes (two Sodium Calcium Silicates and one Lithium Barium Silicate) at the left of *B*.

selectivities for collodion films plated on glass following Ling and Kushnir (1961) or on silver are identical not only in pattern but also in magnitude to those for glasses having the corresponding Na^+ to K^+ selectivity. The principle difference between glass and collodion appears in a somewhat higher selectivity of the collodion systems for H^+ relative to the cations (probably the result of the higher polarizability to the expected for the carboxylic group).

Since the *pattern* of the glass electrode's selectivity appears to be constant for various compositions, it should be possible to simplify matters still further by defining quantitatively the effects of substituting various components of the glass. Recall that Figures 4, 6, and 7 demonstrate that the Na^+ to K^+ selectivity depends in essence only on the ratio of Na^+/Al^{+++} (or Na_2O/Al_2O_3) in a sodium alumino-silicate glass. This relationship can be made more explicit by replotting the Na^+–K^+ selectivity as a function of Na^+/Al^{+++} ratio as has been done in the solid curve labelled NAS in Figure 10. Notice that over an important range of composition and selectivity there is a linear relationship between Na^+/Al^{+++} ratio and log $K_{n_{NaK}}$; expressible by equation

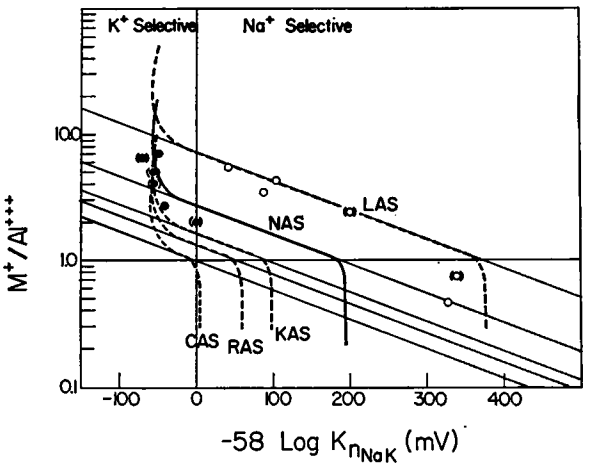

FIGURE 10 Relationship between Na^+ — K^+ selectivity and alkali cation to Aluminum ratio in glasses. The solid curve plots the empirical relationship between $K_{n_{NaK}}$ and Na^+/Al^{+++} (at constant 50% Si^{++++}) extracted from the data of Fig. 6. The dashed curves correspond to the relationship expected (see text) for the Li^+, K^+, Rb^+, and Cs^+ analogues (labelled LAS, KAS, RAS, CAS respectively). Experimental data for Li_2O—Al_2O_3—SiO_2 glasses are plotted as open circles, while data for K_2O—Al_2O_3—SiO_2 glasses are plotted as filled circles. Parentheses around data points denote compositions for which the M^+/Al^{+++} ratio is known at present only in terms of their "batch" or initial compositions rather than the usual final, chemically analyzed compositions. The fine straight lines correspond to the linear equation (4) and have the same slopes for all glasses, differing only in their intercepts. The vertical line indicates the separation between K^+ selective and Na^+ selective compositions. From the $K_{n_{NaK}}$ value of this figures one can predict the approximate selectivities for the other alkali cations and H^+ by using Fig. 9B for K_n or 8A for K' at pH 7.

(4) $$\log K_{n_{Na/K}} = 7.42 \log \frac{Na^+}{Al^{+++}} - 3.2$$

This dependence of $K_{n_{Na/K}}$ upon Na^+/Al^{+++} ratio is to be expected under the postulate that the electrostatic field strength of the $(AlOSi)^-$ site (defined in section 3) governs selectivity since every Na^+ in excess of Al^{+++} will "screen" the $(AlOSi)^-$ site so as to lower its effective field strength. Certain specific consequences of the model propounded in section 3 which are apparent in Figure 10 are: (1) For all glasses in which Na^+/Al^{+++} exceeds 1, the greater the ratio of Na^+/Al^{+++}, the lower the field strength and consequently the less preferred should Na^+ be relative to K^+ and also the lower should be numerical value of the observed selectivity rank order. (2) However, for all Na^+/Al^{+++} ratios less than 1, selectivity should remain constant since the properties of the individual $(AlOSi)^-$ sites no longer change with the changing Na^+/Al^{+++} ratio; for there is then only one Na^+ per $AlOSi^-$ (the excess Al atoms no longer achieving tetrahedral coordination to produce sites of the requisite field strength for cation selectivity). For this reason the Na^+/K^+ selectivity approaches a limit of 2000 : 1 as Na^+/Al^{+++} becomes less than 1, which causes the deviation from equation 4 at the right of Figure 10.

On the other hand, the glasses are observed grossly to imbibe H_2O and become "water-swollen" as the ratio of Na^+/Al^{+++} becomes very high. This effect, as discussed in section 3, would be expected to produce a diminution in selectivity *magnitude* at a given field strength and leads to the maximum of K^+/Na^+ selectivity of 20 : 1 observed here (and also to the maximum in K^+/Na^+ selectivity in Figures 8 and 9).[13]

While the $(AlOSi)^-$ group is the same in any alkali alumino-silicate, the screening effects for the various alkali cations should be different for each M^+ cation (cf. Fajans, 1931, 1957, 1959; Weyl, 1951a, 1951b, 1956). Let us therefore postulate that the only difference among all alkali alumino-silicates will be that a different ratio of M^+/Al^{+++} will be needed to produce a given field strength (and hence a given selectivity). This leads to the prediction that not only does the ratio Na^+/Al^{+++} determine the selectivity of sodium alumino-silicates, but more generally the ratio M^+/xAl^{+++} determines the selectivity of cation responsive glasses of this type.[14] Thus, all Al^{+++} containing glasses having the same ratio M/xAl would be expected to have the same selectivity. If x is taken as 1 for sodium alumino-silicates, then examination of a number of representative lithium alumino-silicates and potassium alumino-silicate (and linear extrapolation of these results to the rubidium alumino-silicates and cesium alumino-silicates using the crystalline radii of Li^+, Na^+, K^+, Rb^+ and Cs^+

[13] The effect of H_2O swelling is possibly complicated in Fig. 10 by the simultaneous appearance of the effects of (SiO^-) sites as the $(AlOSi)^-$ sites become increasingly sparse with increasing Na^+/Al^{+++} ratio.

[14] Still more generally, Al^{+++} can be replaced by any triply charged cation, R^{+++}, in four fold coordination such as B^{+++} or Sc^{+++} and a new set of scaling factors x', and x'' should be all that is needed to characterize these systems.

in the extrapolation) indicate that the values of x should be: Li, 2.5; Na, 1; K, .6; Rb, .5; Cs, .37. To state, this means that when one replaces Na_2O by Li_2O one needs to use only 1/2.5 as many moles of Al_2O_3 to obtain a given set of electrode selectivity properties. On the other hand, if one replaces Na_2O by K_2O, one needs nearly twice as much Al_2O_3 to obtain a given selecitivity. And so forth. These facts, adjusted for the expected water swelling deviations at the left at high M^+/Al^{+++} and for the lack of change of selectivity at the right with M^+/Al^{+++} ratios lower than 1, permit one to predict the dashed curves (LAS, KAS, RAS, CAS) of Figure 10 for the Li_2O, K_2O, Rb_2O, and Cs_2O homologues of the Na_2O—Al_2O_3—SiO_2 glasses, respectively. The extent to which these predictions are satisfactory is indicated by the plotted data points for lithium alumino-silicate and potassium alumino-silicate glasses.

There are several noteworthy consequences of the above considerations. On the one hand, the lithium alumino-silicate system offers both a broadened range of usefully selective Na^+ electrode compositions as well as a much higher magnitude of Na^+ to K^+ selectivity than does the sodium alumino-silicate system. Conversely, the potassium alumino-silicate system offers a broadened range of compositions useful for K^+ detection, although the maximum K^+ to Na^+ selectivity remains of the order of about 20 to 1. It is to be expected that the rubidium alumino-silicates and cesium alumino-silicates will offer still further broadened regions of K^+ selectivity and possibly even some increase in the magnitude of K^+ to Na^+ selectivity.

The Dependence of the Non-Ideal (n) Behavior of the Electrodes on the Field Strength

The characterization of cation responsive glass electrodes should not be concluded without at least pointing out that not only is n a function of composition as has been indicated in Figure 5, but that it can be expressed as a *dependent* function of $K_{_{Na}}$. This is indicated in Figure 11 where the relation between n for hydrogen-cation mixtures (n_{HB}) is plotted as a function of $K_{_{NaK}}$ for the Na_2O—Al_2O_3—SiO_2 glasses of Figure 9b. Despite some scatter of the data, it is clear that a systematic relationship exists between n and $K_{_{NaK}}$ which is capable of analytical expression and is of great theoretical interest as is discussed elsewhere (Eisenman, 1961a, c). I will only comment here that comparison of Figures 11 and 9b indicate that n_{HB} is a function of the *divergence* of the selectivity isotherms of the various cations in Figure 9b relative to H^+, such that when the isotherms are parallel (as with Na^+ and H^+), $n = 1$. The more strongly do the isotherms diverge, the larger is the value of n (cf. H^+-K^+, for example). On the other hand, n also depends upon the *magnitude* of the selectivity difference, so that to the right of Figure 11, where selectivities are small (see Fig. 9b in this region) n tends to approach 1 for all cations.

The above implies that the behavior will be ideal (i.e. n will equal 1) between those ions whose selectivity isotherms of Figure 9b are parallel at the field strength of the glass in question and also that the greater the divergence of the isotherms and

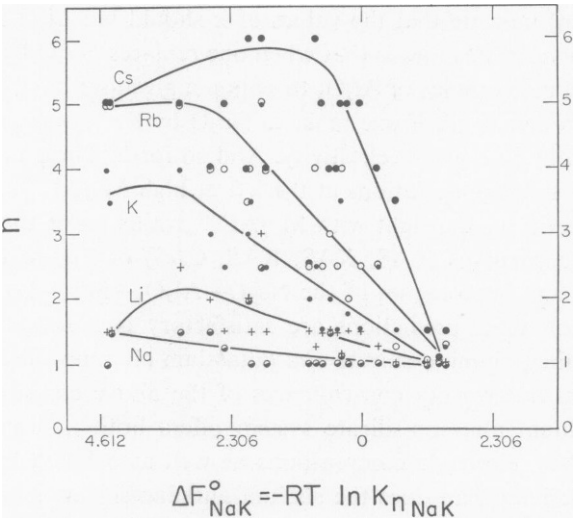

FIGURE 11 Values of n for the Na_2O—Al_2O_3—SiO_2 glasses of Fig. 9*B* plotted as a function of K_{nNaK} in the same manner as Fig. 9*B*. The solid curves represent visually estimated averages of the experimentally observed values.

the greater their energetic separation, the more non-ideal will be the behavior observed (i.e., the larger will be the value of n).

3. THE MODE OF OPERATION OF THE GLASS ELECTRODE

The Origin of the Potential

It should be apparent that the usual pH responsive glass electrodes are but extreme members of the family of cation responsive glasses. Therefore all particular considerations of the theory of the H^+ glass electrode are relevant to the general question of the mode of operation of cation responsive glass electrodes. The theories of the H^+ electrode have been summarized extensively by Dole (1941, see in particular his excellent review in 1931). In the present author's opinion they fall into two broad classes.[15] In one view, H^+ is supposed to be the most mobile cation in the glass and therefore to govern the electrode potential through a "diffusion" mechanism (see, for example, Cremer (1906), Michaelis (1926), Quittner (1928), Dole (1931), Nagasawa and Kobatake (1952) and the highly pertinent papers by Teorell (1935, 1937, 1951),

[15] For the present treatment it is unimportant to distinguish as Dole does between affinities arising from undefined, non-specific interactions with the glass "phase" as opposed to affinities resulting from interaction with specific ion exchange sites. I have therefore lumped Dole's (1931) categories of "phase-boundary" and "ion exchange" theories into one class of equilibrium phenomena. A similar situation exists with regard to permeability; for it has been recognized by Wilbrandt (1934) and Teorell (1953) that there is no fundamental distinction to be drawn between "solubility membranes" and "fixed charge membranes."

Meyer and Sievers (1936), and Sollner (1945a, b; 1958)). In the other, the electrode potential is considered to be a typical phase boundary potential in which the affinities of the various ions for the glass as opposed to their affinities for water are important in what amounts to an ion exchange equilibrium (see for example, Horovitz (1923, 1925), Schiller (1924), Lengyel (1931), Dole (1934, 1941), Nicolskii (1937), Tendeloo (1947) and also the formally similar considerations of Haber and Klemensiewicz (1909), Michaelis and Fujita (1924) Gross and Halpern (1925a, 1925b, 1934) and Hughes (1928), and the pertinent recent work of Ling (1961)). In neither view has an attempt been made as yet to explain, in terms of elementary properties of the ions and glass atoms involved, how either the postulated greater mobility or affinity for H^+ arises (nor, for that matter, how the mobility or affinity might be altered to produce alkali cation selective glasses on the one hand or H^+ selective glasses on the other). More generally stated, there is no atomic explanation of the specificity of the glass, whether it be the specificity implicit in the classical problem of H^+ vs. the group 1A cations for the pH glass electrode or the more recently discovered specificity among the members of Group 1A. *It is to the question of specificity that consideration is given here and no attempt will be made to resolve, between diffusion and phase boundary, the origin of the glass electrode potential.* However, it should be remarked that the present results are consistent with the praticularly simple phase boundary view.[16] The reader's attention is called to Teorell's (1953) review of transport and electrical phenomena in ionic membranes which is highly pertinent to the diffusion-phase boundary question. The recent analyses of Helfferich (1959) and Mackay and Meares (1960) perhaps offer a means of resolving this question in conjunction with mobility measurements in glass and the activity coefficient considerations of the Appendix of this present paper. *Added in proof:* Studies with G. Karreman have been concerned with this problem and attempt the analysis of the electrode potential origin (Eisenman and Karreman, 1962, Karreman and Eisenman, 1962)).

The Origin of Specificity

What factors are involved in specificity? One can answer with assurance that these must be fundamentally the free energies of interaction of the cations with water on the one hand and with anionic "sites" in the glass on the other. This will be so because the competing interactions of cations with water molecules versus the cation interactions with the glassy sites will control not only the equilibrium ion exchange prop-

[16] In this regard recall that the equations found experimentally to govern electrode potential and equilibrium ion exchange are at least formally identical (Eisenman et al, 1957a). Also note that the selectivity rules governing both ion exchange and potential are essentially identical when the corresponding free energy quantities are plotted (compare figures 8a and 8b). Furthermore, the introduction of Al^{+++} into alkali silicate structures results not only in typical cation exchange materials, but also cation responsive glass electrodes. It should be noted that Marshall (1948) suggested the existence of an important formal relationship between the mobility ratios of cations in a liquid junction (with zero anionic mobility) and the differential heats of adsorption of the cations.

erties of the glass (and consequently the phase boundary potential) but these interactions will also be importantly involved in the activation energies governing cationic migration from the solution into the glass and also the migration within the water-swollen regions of the glass, although the activation energies would be expected to be easily calculable functions of the elementary free energies only in certain limiting situations.

The competing interactions of cations with the glass versus water can be represented by an ion exchange reaction:

$$(8) \qquad I^+ \text{ (Glass)} + J^+ \text{ (aqueous)} \rightleftarrows J^+ \text{ (glass)} + I^+ \text{ (aqueous)} + \Delta F_{ij}^0$$

where I^+ (glass) and J^+ (glass) represent the cationic species I^+ and J^+ in the glass and I^+ (aqueous) and J^+ (aqueous) represent these species in dilute aqueous solution. The standard free energy change of the reaction, ΔF_{ij}^0, governs directly the ion-exchange equilibrium of the glass through equations (9) and (10) and its phase boundary potential through equation (11).[17]

$$(9) \qquad \Delta F_{ij}^0 = -2.303RT \log_{10} K_n = -RT \ln K_n$$

$$(10) \qquad K_n = \frac{(I^+)}{(J^+)} \left[\frac{J^+_{glass}}{I^+_{glass}} \right]^n$$

$$(11) \qquad E = E^0 + \frac{RT}{F} \ln [(I^+)^{1/n} + K_n^{1/n} (J^+)^{1/n}]^n$$

The dependence of the free energy change, ΔF_{ij}^0, of reaction (8) upon the interaction of cations with water vs. glass is written explicitly in equation (12) in which $\bar{F}_{I^+}^{hyd}$ symbolizes the partial molal free energy of hydration of the ion I^+ while $\bar{F}_{I^+}^{glass}$ represents the partial molal free energy of interaction of I^+ with the glass.

$$(12) \qquad \Delta F_{ij}^0 = \bar{F}_{I^+}^{Hyd} - \bar{F}_{J^+}^{Hyd} + \bar{F}_{J^+}^{glass} - \bar{F}_{I^+}^{glass}$$

What do we know of the energies? Those with water ($\bar{F}_{I^+}^{Hyd} - \bar{F}_{J^+}^{Hyd}$) are constants characteristic of each cation and are known experimentally with great accuracy. Their values extracted from the tabulation of Rossini, et al. (1952) and referred to Cs^+ are given in Col. 3 of Table 1. On the other hand, the free energies of interaction with the glass ($\bar{F}_{J^+}^{glass} - \bar{F}_{I^+}^{glass}$) will depend upon its chemical composition and will be evaluated below.

Our specific problem here thus becomes one of characterizing the affinities of the various cations for different glasses as a function of the chemical composition of the glass and combining these with the cation affinities for water in order to solve for the free energy change, ΔF_{ij}^0, of equation 8. More generally, for the analysis of equilibrium

[17] The relation of these equations to the more usual "ideal" equilibria in which $n = 1$ is discussed briefly by Eisenman, et al. (1957a) and more extensively by Eisenman (1961c). From the assumptions under which equations 10 and 11 were derived, K_n should be identical to the thermodynamic equilibrium constant. See Appendix.

specificity in any system in which the basic processes involve cation exchange, all that is necessary is the evaluation of equations of the type of equation 12. Furthermore, free energy considerations similar to those implicit in equation 12 underlie the specificity of such non ion exchange phenomena as the activity coefficients, osmotic coefficients and solubilities of concentrated solutions of strong electrolytes and the association of weak electrolytes.

Two complementary approaches to the analysis of equilibrium specificity are apparent. In one, entirely empirical methods can be used to evaluate the free energies of equation 12 or their counterparts for the specificity of other processes (cf. the definition of such processes in Figure 2 of Eisenman, 1960a). In this way it is possible, in the total absence of theoretical considerations, to understand a great deal about the specificity of a system such as the present glass electrode through the use of "halide" models for its sites of interaction with cations. One need only ask, "what specificity properties would the various halide anions manifest for Na^+, K^+, H^+, NH_4^+, Ag^+, or Ca^{++}, etc., in competition with water?" This question can be answered directly from thermo-chemical data readily available for the halides and for the cations of interest in a variety of states (Rossini, et al., 1952). Thus, for example, if we assumed that the sites in various kinds of glass could be represented by halide crystal models, we could write reaction 12 as,

$$\text{(12a)} \qquad \Delta F_{ij}^{0} = \bar{F}_{I^+}^{\text{Hyd}} - \bar{F}_{J^+}^{\text{Hyd}} + \bar{F}_{J^+}^{\text{cryst}} + \bar{F}_{I^+}^{\text{cryst}}$$

ΔF_{ij}^{0} could then be evaluated from the experimentally known free energies of formation, ΔF_f^{0} [18] of the various halide crystals of I^+ and J^+ in combination with the free energies of formation of I^+ and J^+ in dilute aqueous solutions. For a "chloride type" site equation (12b) yields the desired ΔF_{ij}^{0}, while analogous equations yield the energy for Br^-, I^-, or F^- sites (as well as for molecular anions of interest such as carboxyl, phosphate, and sulphate).

$$\text{(12b)} \qquad \underset{\text{(chloride)}}{\Delta F_{ij}^{0}} = \Delta F_f^{0} I^+(\text{aq., Hyp., 1.0M}) - \Delta F_f^{0} J^+(\text{aq., Hyp., 1.0M}) + \Delta F_f^{0}\ \text{J Cl (crystal)} - \Delta F_f^{0}\ \text{I Cl (crystal)}$$

On the other hand, it is also possible to evaluate the energies of equation (12) theoretically from considerations of the elementary properties of the atoms involved. This constitutes an atomic analysis of what in the first method is an essentially thermochemical approach and provides the ultimate insight into the mechanism of origin of specificity. Nevertheless, it should be emphasized that the *rules* of specificity as a function of anionic properties can be derived by the first method quite independently of such atomic theoretical considerations.[19]

It will be seen below that in certain simple limits such theoretical calculations are

[18] The terminology of Rossini, et al. 1952 is used here and in equation (12b).

[19] In this regard cf. the essentially identical specificity patterns developed in the entirely experimental Fig. 4 and the theoretical Fig. 3 of Eisenman, 1960a.

possible with reasonable rigor; but that in other situations, notably those important systems whose intimate structures contain water and may be called "water-swollen", present theories are rather crude. In these latter situations, reliance is necessarily placed entirely upon the empirical treatment. For, despite the fact that the positions, numbers, orientations, and freedom of motion of the water molecules, counterions, and sites are in principle calculable since they depend upon elementary properties of sites, counterions, water molecules, and other structural elements of a system, it is not possible at present to solve rigorously for these quantities in an extended three dimensional system.[20]

The Affinities of Cations for the Sites in H *Responsive and Alkali Cation Glasses*

The feature of importance of a glass (or of a protein or lipid, for that matter) as far as its interactions with cations is concerned is that it is a structure in which negatively charged oxygens are polymerized upon a framework of highly positively charged atoms such as Si^{++++} and Al^{+++}. A glass is therefore very similar to a biological oxygen polymer (such as the polymers of phosphate and carboxyl groups in protoplasm and cellular membranes).[21] In both cases the nearest neighbor to the cation is the oxygen and it is only the "effective field" of this oxygen as modified by the more distant C, P, Al, or Si which is seen by the cations. Also, in its interaction with cations, a glass appears to be in no fundamental respect different from a monomeric oxyanion such as a sulphate, phosphate, or silicate in aqueous solution, nor does the lack of long range order make a glass importantly different from a crystalline polymer.

The atomic structure of the site of cation exchange in the usual H^+ responsive glasses, which are generally alkali silicates, is diagrammed at the left of Figure 12 and will be referred to as an SiO^- type site. Such a site would occur by hydrolysis at the surface of pure SiO_2. It also occurs wherever a lattice breaking alkali cation exists in an alkali silicate glass or crystal. The alkali cations in such a glass migrate by exchange diffusion from one such site to another (Quittner (1928), Anderson and Stuart (1954) and Isard (1959); cf. also Sollner's important papers (1945), 1949a,

[20] The solution of this important problem will require a completely particulate analysis of factors so far only approximated in Bjerrum's (1926) theory of ion pairs, Scatchard's (1932, 1936) theory of concentrated solutions of alkali halides, Gurney's (1953) microscopic but only partially quantitative considerations of ion size and water structure, and Frank's and Thompson's (1959) diffuse lattice cloud treatment of concentrated solutions. However, an important step has been taken toward a solution by Ling's (1960a, 1961) completely particulate treatment of site, counterion, and water molecules in a linear array. Hopefully, development of a microscopic theory of the lattice energies of the crystalline hydrates of the alkali halides, being initiated in collaboration with G. Karreman, will be capable of extension to a semi-crystalline treatment of solutions of electrolytes and to water-swollen ion exchangers. A clear recognition of the importance of many of the above factors is given by Katchalsky (1954) in his noteworthy review of polyelectrolyte gels.

[21] See Katchalsky's (1954) comprehensive review of polyelectrolyte gels for an extensive discussion of biological polymers.

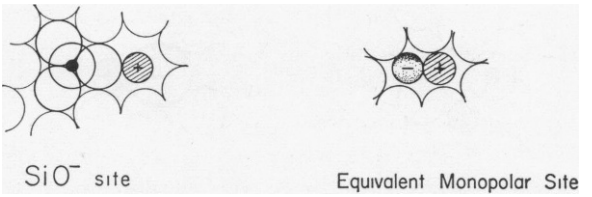

FIGURE 12 Three dimensional diagram of an SiO^- site and its counterion: in an alkali silicate glass. At the left is drawn a typical tetrahedral SiO^- site and counter ion surrounded by adjacent oxygens of the glass (partial circles). The large circles represent oxygens (the upper one projects toward the reader). The small dark circle represents Silicon, which lies at the center of the four oxygens, while the shaded cation represents Na^+. All species are drawn to a scale corresponding to their ionic crystal radii. At the right is diagrammed to the same scale the singly charged anionic site equivalent to SiO^- as discussed in the text. Note the essentially "anionic" character of the environment of the Na^+. Also note that H_2O molecules would be roughly the same size as these oxygens, being merely oxygens with two protons imbedded therein and oriented away from the Na^+. Finally, note that the site diagrammed here is geometrically similar to the $H_2PO_4^-$ ion or a phosphate site in a phosphoprotein, phospholipid, nucleic acid, or ATP molecule, although important field strength differences among these, of course, exist.

1949b, 1958) regarding the analogous problem in perm-selective membranes and also Teorell's (1953) superb review of transport processes and electrical phenomena in ionic membranes). They are also exchangeable for cations from an aqueous solution into which the glass is dipped (Haugaard, 1941). On the other hand, the atomic structure of the site in alkali cation responsive glasses, which are typically alkali silicates in which a triply charged ion such as B^{+++} or Al^{+++} has replaced some of the Si^{++++} in fourfold coordination, is illustrated in Figure 13 and will be referred to as an $(AlOSi)^-$ type site. As long as the ratio of M^+/Al^{+++} in the glass exceeds 1, each Al^{+++} enters into tetrahedral coordination[22] restoring the glass lattice and forcing one alkali cation from its "lattice breaking" position in the alkali silicate glass into an "interstitial" position diagrammed in the alkali alumino-silicate glass.[23]

I shall first examine what differences in interaction with cations may be expected

[22] Evidence for the tetrahedral coordination of Al^{+++} has been obtained by infra red absorption by Moore and McMillan (1956) as quoted by Lengyel and Csakvari (1960). It is consistent with the generally accepted existence of such a structure in the alkali aluminosilicate mineral counterparts of the glasses. Furthermore, a direct characterization of the tetrahedral coordination of B^{+++} by Silver and Bray (1958) in alkali borate and borosilicate glasses by nuclear quadrupole resonance is important since it indicates that the same coordination should occur in alkali aluminosilicates.

[23] The initial suggestion of the author and his colleagues (Eisenman, et al. (1957c) summarized by Friedman (1961)) for the effect of Al^{+++} or B^{+++} introduction into the glass incorrectly considered the affinities of the ions for lattice breaking sites, AlO^-, of a type similar to SiO^- of Figure 12 rather than for the present "interstitial" $(AlOSi)^-$ sites of Figure 13. Such an AlO^- site, which *does not* exist in these glasses, would on present grounds be expected to be even more H^+ preferring than SiO^-.

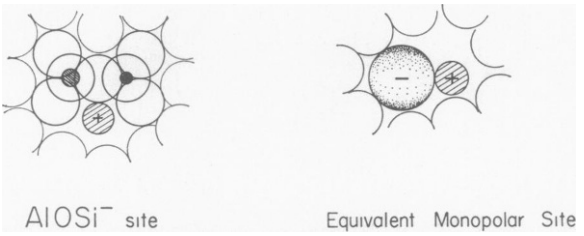

FIGURE 13 Diagram of a typical AlOSi⁻ site and its equivalent monopolar site constructed in the same manner and to the same scale as Fig. 12. The darkly shaded Al of the left hand tetrahedron will be seen to be slightly larger than the Si of the right.

from detailed models of particular sites. Then, in the next section, I shall attempt to analyze in a more general manner the salient differences between these sites in terms of what will be defined as their "equivalent anionic field strength." This parameter will be shown to be susceptible to empirical assessment as well as theoretical calculation.[24]

Let us begin by considering the situation for the SiO^- site of a typical H^+ selective glass. The interactions of the various cations with such a site will depend upon a number of elementary atomic forces and are calculable precisely only if one takes into account all forces between the exchangeable ions and the neighboring atoms of the glass. It is not possible to do this with complete rigor at present and I shall therefore evaluate only the simplest electrostatic forces here.[25] Even in such a simple model the serious problem remains of how to assign the charges. Here we will examine an extreme case in which it is assumed that each oxygen completely captures 2 electrons from the more electropositive elements and that a proton is representable by a single positive charge buried 0.4 Å within the oxygen. (However the general conclusions would also be true for considerably less extreme charge distributions).

The resulting multipolar model for an SiO^- site is represented by the silicate anion $H_3SiO_4^-$ in Figure 14 upper for which the energies of interaction with rigid sphere

[24] The field strength of the sites will be treated here as if it were a continuously variable quantity, as I think likely in the present glass system. However it is to be emphasized that there will also be in nature systems composed of mixed populations of sites having discreetly different field strengths, whose behavior would mimic that of sites of continuously variable field strength. The distinction between these two situations, while important in its own right, is not crucial for the present analysis since the same basic principles of ion-site vs. ion-water interaction govern the specificity manifested by the sites in either case.

[25] The reader should not be misled by the quantitative nature of the following calculations which are to be taken as representing at most a semi-quantitative accounting of the principle factors involved and are but a first step in the rigorous treatment. In this regard, cf. Figs. 5a and 5b of Eisenman (1960a) which indicate the extent to which simple electrostatic forces alone account for the energies of interaction of the alkali cations with halide anions not only in the symmetrical crystalline situation, but also in the highly assymetrical diatomic gaseous molecules. Also note the extent to which electrostatic forces account for the bulk of the hydration energies of the alkali cations in Bernal and Fowler's (1933) theory.

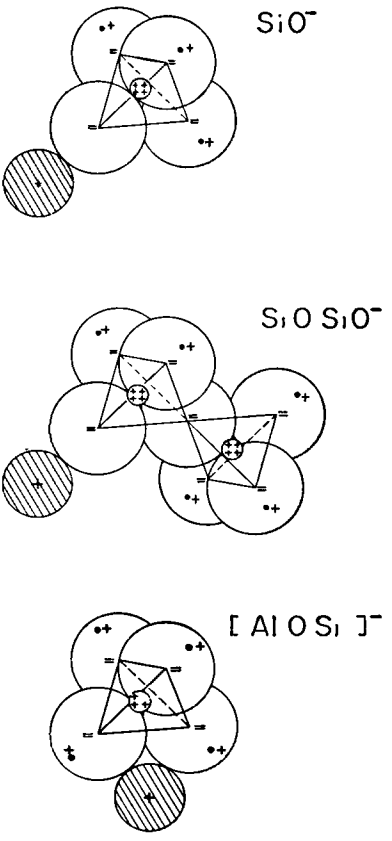

FIGURE 14 Diagrams of extreme ionic models for SiO^-, $SiOSiO^-$, and $AlOSi^-$ sites used in calculating the energies of Tables 1–3 as described in the Text. The groups are represented by their monomers $H_3SiO_4^-$ and $H_4AlO_4^-$ and the dimer $H_5Si_2O_7^-$. The large spheres are oxygens. The shaded sphere represents a cation. The small spheres represent Si^{++++} or Al^{+++}, and the dots represent protons. The two negative charges of the oxygens are located at the corners of the tetrahedron indicated by lines between the oxygen centers, and the protons (indicated by small dots and + charges) are considered as located in their expected minimum energy positions 0.4 Å beneath the surface of the oxygens in H_4SiO_4, $H_6Si_2O_7$, and $H_4AlO_4^-$.

cations having Pauling crystal radii were calculated at their minimum energy position in collaboration with G. Karreman (as were also the other models in Figure 14), the proton being treated as a cation of radius −0.4 A. The resulting electrostatic energies of interaction are indicated in the first column of Table 1; while the second column of Table 1 gives the difference of these energies referred to Cs^+. The third column lists the differences of experimental free energies of hydration referred to Cs^+, which as mentioned before are constants for the exchange of equation 8. Column 4 lists the differences between columns 2 and 3 and is the solution for ΔF_{ij}^0 of equation 12 under the assumption that the free energy differences within the glass are predominantly determined by internal energy contributions.[26]

[26] The justification of using a dielectric constant of 1 in the present calculations as well as neglecting the possibility of water molecules interposed between cation and site is a consequence of the finding discussed later that the effect of H_2O (or other molecules) in the vicinity of the site is one affecting magnitude but not sequence of selectivity. In this regard, the above calculations are valid in indicating H^+ preference over alkali cation although the magnitude represents only a limiting value.

TABLE I
SiO^- SITE

	(1) Electrostatic Energies with SiO^- site (KCal/mole)	(2) Differences of Electrostatic Energies referred to Cs^+ (KCal/mole)	(3) $(\bar{F}_{I^+}^{hyd} - \bar{F}_{Cs^+}^{hyd})$ Experimental* (KCal/mole)	(4) ΔF_{ij}^0 (KCal/mole)	(5) r_- (A°)
H^+	−493.2	−359.1	−192.673	+166.427	1.07
Li^+	−220.4	− 86.3	− 54.299	+ 32.001	.91
Na^+	−183.2	− 49.1	− 30.393	+ 18.707	.86
K^+	−153.8	− 19.7	− 12.745	+ 6.955	.83
Rb^+	−145.3	− 11.2	− 7.679	+ 3.521	.81
Cs^+	−134.1	0	0	0	.79

* After Rossini, et al. (1952)

These ΔF_{ij}^0 differences (referred to Cs^+ as J^+) indicate a clear preference by the model SiO^- sites for H^+ over all the group Ia cations. They also indicate that the sequence of preference of group Ia cations is $Li^+ > Na^+ > K^+ > Rb^+ > Cs^+$. A glass or mineral with SiO^- sites would therefore be expected to behave as a highly H^+ selective electrode at least as far as its phase boundary potential is concerned. It would also be expected to exhibit a marked ion-exchange preference for H^+. Both of these expectations are in accord with observation.

On the other hand consider the $(AlOSi)^-$ site in a typical cation responsive glass. The energies of interaction of cations with such an $(AlOSi)^-$ type site have been calculated from Figure 14 lower for an hypothetical "aluminate" ion, $H_4AlO_4^-$ using the same assumptions as above. The results are given in Table 2. However, only an approximate upper limit of the interaction energy with the $(AlOSi)^-$ group for H^+ is indicated here because the minimum energy position for the H^+ has not yet been determined precisely. In constrast to the situation with SiO^-, the ΔF_{ij}^0 values in column 4 of Table 2 for $(AlOSi)^-$ indicate a clear preference for the alkali cations over H^+ and also indicate a selectivity rank order among the group Ia cations ($Cs^+ > Rb^+ > K^+ > Na^+ > Li^+$) opposite to that of the SiO^- site! A glass or mineral with such $(AlOSi)^-$ sites would therefore be expected to function as a cation selective electrode (at least for the phase boundary potential) and also as a good cation exchanger under the usual ionic concentrations. Both expectations are fulfilled.

While it is possible to extend the above calculations to assess the role of adjacent oxygens, silicons, and "screening" cations, the lack of detailed knowledge of their positions makes it more profitable at present to simplify the above representations by a singly charged equivalent monopolar site, variations in whose "effective field strength" due to screening (cf. for example, Fajans (1931, 1957, 1959) as extended by Weyl (1951, 1956)), etc. can be represented as simple variations of its radius,

TABLE II
(AlOSi)$^-$ SITE

	(1) Electrostatic Energies with $(AlOSi)^-$ site (KCal/mole)	(2) Differences of Electrostatic Energies referred to Cs^+ (KCal/mole)	(3) $(\bar{F}_{I^+}{}^{hyd} - \bar{F}_{Cs^+}{}^{hyd})$ Experimental* (KCal/mole)	(4) $\Delta F_{i,j}{}^0$ (KCal/mole)	(5) r_- (A°)
H^+	α −200	−105.3	−192.673	−87.373	α 2.0
Li^+	−123.1	− 28.4	− 54.299	−25.899	2.1
Na^+	−115.3	− 20.6	− 30.393	− 9.793	1.93
K^+	−104.3	− 9.6	− 12.745	− 3.145	1.85
Rb^+	−100.2	− 5.5	− 7.679	− 2.179	1.83
Cs^+	− 94.7	0	0	0	1.82

* After Rossini, et al. (1952)

r_-.[27] In this way we can symbolize the effective field strength of a wide variety of oxyanion systems by what amounts to a "halide type" model for the oxyanion, which not only simplifies the theoretical calculations but also allows the use of data from the alkali halides in appropriate states as "models" for oxyanion systems.

[27] For example, the above calculations were for isolated sites. It is fairly easy to calculate the energies for the dimer of silicic acid by using the charge distributions indicated in the middle diagram of Figure 14, but it becomes very difficult to assess the minimum energy positions of the cations for higher polymers. Nevertheless, it seems apparent that the predominant effect of replacing the protons of silicic acid by additional silicate tetrahedra is one of increasing the effective negative charge on the site (predominantly through the effects of additional oxygens which are thereby brought into the vicinity of the site). In this regard notice that the calculations for the disilicate anion of Table 3 indicate no important effect from replacing a distant proton by a silicate tetrahedron, which is in accord with the experimental observation that the first dissociation constants for silicic acid and disilicic acid are identical (K_a being $10^{-9.8}$ in both cases (Iler, (1955), p. 22)). This finding supports the validity of the simple models of SiO^- and $(AlOSi)^-$ of Figure 14 in which adjacent silica tetrahedra are replaced by protons for ease of calculation, the SiO^- site being approximated by the $Si(OH)_3O^-$ ion while the $(AlOSi)^-$ site is approximated by an $Al(OH)_4^-$ ion. On the other hand, the first dissociation constant of the trisilicate anion is considerably smaller (K_a being $10^{-12.8}$ (Iler (1955) p. 22)) which indicates a stronger negative charge acting on the H^+ in this case. The same effects in a completely polymerized silicate should result in an even stronger effective negative charge on the site. In opposition to the above effects, the presence of alkali metal oxides above the ratio $M_2O/Al_2O_3 = 1$ will result in SiO^-M^+ structures in the vicinity of the $(AlOSi)^-$ site which would be expected to decrease the net field strength either through direct electrostatic effects or by "screening". It is for this reason that selectivities of the glass are expected to depend upon the M^+/Al^{+++} ratio in the glass. It is also for this reason that the various alkali cations would be expected to produce different field strengths for a given M^+/Al^{+++} ratio in the glass and thereby different particular selectivities for each glass while preserving the overall selectivity pattern as illustrated in Figure 9. In this regard, note that the effects of replacing Na^+ by Li^+, K^+, Rb^+, or Cs^+ are in the direction to be expected from the increased screening with increasing atomic number. They are also consistent with a simple electrostatic model for this effect (Eisenman, 1961a).

TABLE III
SiOSiO⁻ SITE

	(1) Electrostatic Energies with $SiOSiO^-$ site (KCal/mole)	(2) Differences of Electrostatic Energies referred to Cs^+ (KCal/mole)	(3) $(\bar{F}_{I^+}^{Hyd} - \bar{F}_{Cs^+}^{Hyd})$ Experimental* (KCal/mole)	(4) $\Delta F_{i^+}^0$ (KCal/mole)	(5) r_- (A°)
H^+	−494.5	−360.9	−192.673	+168.227	1.07
Li^+	−220.9	− 87.3	− 54.299	+ 33.001	.91
Na^+	−183.0	− 49.4	− 30.393	+ 19.007	.86
K^+	−153.3	− 19.7	− 12.745	+ 6.955	.83
Rb^+	−144.8	− 11.2	− 7.679	+ 3.521	.81
Cs^+	−133.6	0	0	0	.79

* After Rossini, et al. (1952)

Such equivalent "halide type" monopolar sites are indicated diagramatically to the right of the multipoles of Figures 12 and 13. These are, of course, only approximations of the actual sites but serve the function of relating the present material to the conclusions of a general analysis of specificity developed in initial form in collaboration with J. U. Casby and D. O. Rudin (Eisenman, et al. 1957a, Rudin and Eisenman, 1959). This analysis has been considerably extended by Eisenman and by Rudin and will be published elsewhere (Eisenman, 1961b; Rudin, 1961). However, the principal methods, assumptions, and a number of relevant results of the present author's analysis have been summarized (Eisenman, 1960). It should also be noted that Ling (1960, 1961) has independently carried out a most important analysis of the question of specificity using somewhat different assumptions which agrees in certain salient features with the present author's conclusions.

The "equivalent radius" of a site may be defined as that value of r_- which yields the same electrostatic energy of interaction for each cation as does the complete multipole. Values of these radii are listed in Column 5 of Tables 1–3. For the SiO^- site they are of the order of 0.9 Å ± 0.2 Å, while for $(AlOSi)^-$ they are considerably larger, being of the order of 1.95 ± 0.15 Å. Clearly, the SiO^- site has a much higher effective field strength than the $(AlOSi)^-$ site. It is this difference of field strength which accounts for the gross difference of H^+ vs. cation selectivity between these types of glasses as well as which accounts for the particular pattern of selectivity among the various alkali cations (as should become clear in the following section). However, the effective field strength in the glasses will, of course, be adjustable around these values of r_- as a function of such factors as the aforementioned screening by excess alkali cations in the glass or the spacing between "sites."

To understand the detailed effects to be expected from these factors we must now analyze the dependence of ionic specificity upon anionic field strength of the sites. However, before doing so I should like to note that it is possible to reach the

above important conclusions that SiO^- sites have a much higher field strength than $(AlOSi)^-$ sites in at least two other ways. For example, application of Pauling's Electrostatic Valence Rule (1948, p. 384–386) to the SiO^- site indicates that the oxygen has a residual binding strength for cations of 1 which is markedly stronger than the residual strength of 1/4 of the $(AlOSi)^-$ site. This difference is recognized by Pauling as underlying the weak acid behavior of SiO^- and the strong acid behavior of $(AlOSi)^-$. It is also possible to deduce that $(AlOSi)^-$ will have a much lower field

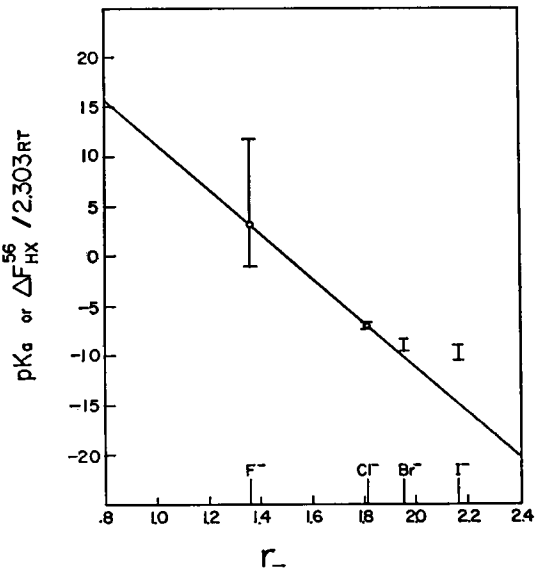

FIGURE 15 Empirical relationship between r_- and pK_a. The straight line determined by the observed pK_a's for HCl and HF and their crystal radii (indicated by the open circles) defines an approximate empirical relationship between r_- and pK_a. This line, represents equation (13) and yields an approximate equivalent r_- for any anion whose pK_a is known experimentally (and vice versa). (Some examples of this are given in Fig. 7 of Eisenman, 1960a). Plotted for comparison are the ranges of experimental (i.e. thermochemical) values of $\Delta F_{HX}^{56}/2.303\ RT$ (as defined by Eisenman, 1960a).

strength than SiO^- and in fact to assign equivalent r_- values to these sites which agree well with the above values on completely empirical grounds through the recognition that a relationship exists between equivalent field strength and acid dissociation constant of a given site. This has been described briefly elsewhere (cf. Figure 7 and Equations 8 and 9 of Eisenman (1960a)) and is represented in Figure 15, which diagrams the approximate relation between r_- and pK_a expressed in Equation 13.[28]

[28] The validity of Equation (13) is roughly testable for the halides by the extent to which the experimental values of $\Delta F_{HX}^{56}/2.303\ RT$ (defined in Eisenman, 1960, cf. particularly Figure 2) correspond. The range of these values are plotted in Figure 15 for comparison with the line drawn according to equation 13.

Eq. (13) $$r_- = -0.044pK_a + 1.5$$

Equation 13 provides a way to obtain a roughly equivalent r_- for any anion whose pK_a is known. Consider the case of the silicate anion, whose pK_a is 9.8. From equation 13, this anion would have an r_- of 1.08 Å, a value in good agreement with the 0.9 ± 0.2 Å previously calculated. A similar r_- value of 1.08 Å is expected for the disilicate anion (whose pK_a is also 9.8) in comparison with 0.9 ± 0.2 Å calculated. On the other hand, a trisilicate anion has a pK_a of 12.8 corresponding to an r_- of 0.94 Å; and one would expect that further polymerization of silicic acid would further decrease r_- as concluded previously. The empirical assessment of field strength for the SiO^- site is therefore in agreement with the calculations in leading to the conclusion that SiO^- is characterized by a high field strength.

On the other hand, although the acid dissociation constant of $(AlOSi)^-$ is not precisely known, Pauling (1948, p. 396) concludes that it is a very strong acid having a strength approximately the same as perchloric acid (i.e. it should have a very negative pK_a of the order of −8). From equation 17 such a pK_a would correspond to an r_- of 1.9 Å which is in agreement with the value 1.95 ± 0.15 calculated by the model. Examination of Figures 15–17 will indicate that such a site would markedly prefer cations when compared to H^+; and systems containing such sites would therefore be expected to function as cation exchangers having cation selective phase boundary potentials, and (in certain limits) correspondingly high cation permeabilities relative to H^+ as well.

General Conclusions Regarding the Specificity Properties of Monopolar Sites

If we postulate that we can represent the most important energetic features of the SiO^- and $(AlOSi)^-$ sites by a monopolar site having an appropriate radius (r_-), we can generate a number of important relationships between ionic specificity and the anionic field strength of individual site, as well as the dependence of specificity upon spacing between sites and the extent to which a glass become "swollen" by imbibing water. In particular it is possible to show that anionic field strength is the principal factor governing the *sequence* (and also the magnitude) of specificity among the alkali metal cations and H^+. On the other hand, the number of water molecules admitted into the vicinity of the interacting anion and cation will be shown to control predominantly the *magnitude* (but not the sequence) of specificity.

To demonstrate the manner in which these conclusions are reached, let us again solve for the free energy change, ΔF_{ij}^0, of Equation 12. First, recall that the interactions of cations with water are constant, being given by the experimental values for $\bar{F}_I^{hyd} - \bar{F}_J^{glass}$ as discussed above. On the other hand, the only factors governing the interactions of I^+ and J^+ with the glass ($\bar{F}_I^{glass}$ and $\bar{F}_J^{glass}$) are the interactions of the cations with the individual sites and the "state" in which the sites exist in the glass. An appropriate "state" can be chosen to represent various spacings between sites, the extent of fixation of the sites, as well as the presence of neutral but polar

molecules in their vicinity (e.g. H_2O molecules in a water-swollen system, or non-site oxygen atoms of the glassy lattice or of a polypeptide backbone[29]). In the following examples we shall first examine the elementary properties of the site which govern selectivity in various representative constant "states" and then examine the effect of varying the "state" for constant properties of the individual site.

If the sites of a glass are first presumed to be in a state of wide separation and water molecules are also presumed to be excluded from their vicinity, then the free energies of interaction between cation and site needed to solve equation 12 are given in first approximation by the internal energies as calculated by Coulomb's law for rigid sites and counterions.[30]

$$\bar{F}_{I^+}^{glass} \cong -\frac{332}{r_{I^+} + r_-} \tag{14}$$

$$\bar{F}_{J^+}^{glass} \cong -\frac{332}{r_{J^+} + r_-} \tag{15}$$

The energies in (14) and (15) are in KCal/mole when distances are in Å. r_{I^+} and r_{J^+} symbolize the naked radii of the ions I^+ and J^+ and are taken as having the following values in Å: Li^+ (0.78), Na^+ (0.98), K^+ (1.33), Rb^+ (1.48), Cs^+ (1.69), H^+ (−0.4), (Goldschmitt, 1926; Pauling 1948). Since r_- is the only variable governing ΔF_{ij}^0 when equations (14) and (15) are inserted in Equation 12, one can plot ΔF_{ij}^0 as a function of r_- as is done in Figure 16 (which corresponds very nearly to Figure 3c of Eisenman (1960a) differing from it only in the neglect here of the very small entropy differences for the various cations with the site.[31]

$$\text{(16a)} \qquad \bar{F}_{I^+}^{glass} \cong \frac{8}{9}\left(\frac{-332}{r_{I^+} + r_-}\right); \qquad \text{(16b)} \qquad \bar{F}_{J^+}^{glass} \cong \frac{8}{9}\left(\frac{-332}{r_{J^+} + r_-}\right)$$

If, on the other hand, the sites in a glass are assumed to be better represented as being very closely spaced (for example, with 6 sites coordinated around each cation and 6 cations around each site) in the extreme, then the Free Energies of interaction of the cation with the glass will be given by equations (17) and (18).

$$\bar{F}_{I^+}^{glass} \cong 1.56\left(\frac{-332}{r_{I^+} + r_-}\right) \tag{17}$$

[29] The importance of the latter structure has been called to my attention by Ling, and may serve to answer the questions raised in the recent discussion by Conway, Eisenman, and Ling on the question of the selectivity magnitudes to be expected in protoplasm (cf. Conway; 1960; Eisenman, 1960b; and Ling, 1960b). Teorell (1953, p. 306) explicitly recognized the importance of polar oriented groups in his discussion of "Ionic Membranes."

[30] It is assumed in these calculations that the differences of entropies of interaction between the various cations and the model sites are negligible. This is supported by thermochemical data. On the other hand, differences in entropies of hydration are far from negligible. But this problem is circumvented by using experimental values directly for the free energies of hydration of the cations.

[31] If, as a further refinement, one also takes into account the electronic repulsion forces through Born's approximation, then this energy still depends only upon r_ being given by equations (16a and 16b).

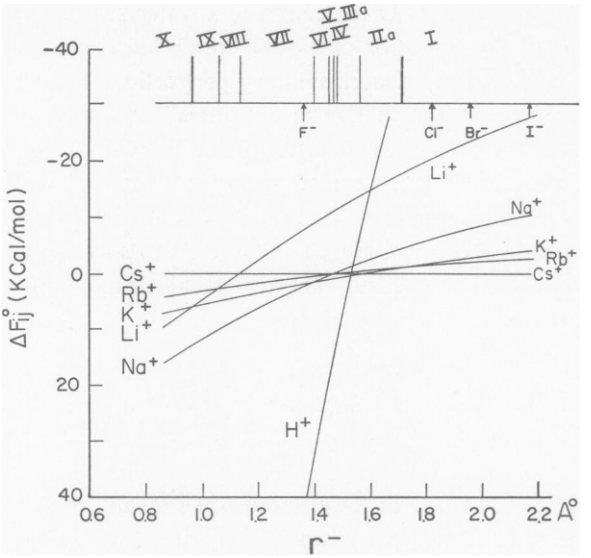

FIGURE 16 ΔF_{ij}° selectivities plotted as a function of r_- for the case of widely separated sites (see text). The roman numerals above the figure indicate the sequences of ionic selectivity pertaining in the regions separated by the vertical lines drawn to the intersections of the various cation isotherms.

$$\bar{F}_{J^+}^{\text{glass}} \cong 1.56\left(\frac{-332}{r_{J^+} + r_-}\right) \tag{18}$$

Equations (17) and (18) are the Born-Landé (1918) equation for the Internal Energy of the alkali halide crystal lattice and also depend only on r_-. Therefore ΔF_{ij}^{0} can be plotted as a function of r_- for closely packed sites in Figure 17 (which corresponds very nearly to Fig. 3D of Eisenman, 1960a).

Finally, if it is desired to consider the state of the site in the glass as complicated by water-swelling, one can take an aqueous solution of corresponding molality as an empirical model for a water swollen exchanger site, as proposed by Cruickshank and Meares (1957). In this case the free energy change of reaction (12) has been shown by Cruickshank and Meares to be given directly from the experimentally known osmotic and activity coefficients by equation (19).

$$\Delta F_{ij}^{0} = 2RT\left(\theta_{IX} - \theta_{JX} + \ln\frac{\gamma_{JX}}{\gamma_{IX}}\right) \tag{19}$$

Despite the fact that (19) represents experimental data, and does not therefore contain r_- explicitly in it, one can show that selectivity at a constant water swelling depends essentially only upon r_- by plotting the free energies as a function of r_- (locating the Halide anions at their crystal radii (r_{F^-} = 1.36 Å, r_{Cl^-} = 1.81 Å,

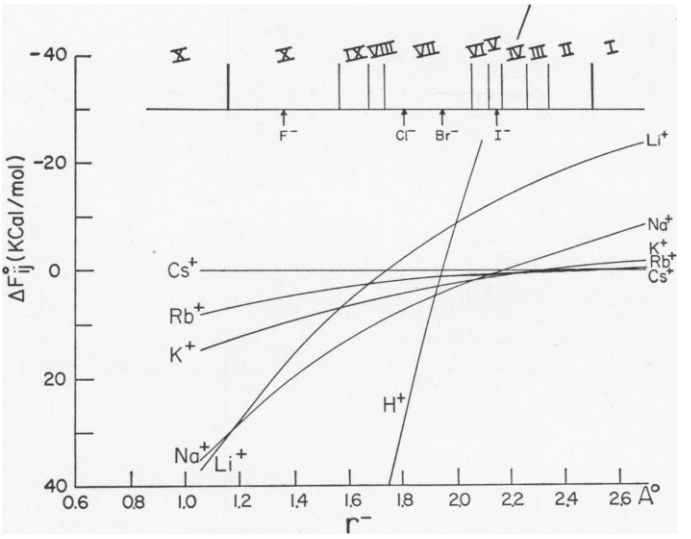

FIGURE 17 $\Delta F_{i,j}^{\circ}$ selectivities as a function of r_- for closely spaced sites. Plotted in the same manner as Fig. 16.

$r_{Br^-} = 1.95$ Å, $r_{I^-} = 2.16$ Å), as has been done in Figure 18 for four widely differing molalities.[32]

Examination of Figures 16–18 yields the following conclusions.

(1) The selectivity among cations in each of these states depends upon $r-$ (i.e., upon anionic field strength).

(2) A particular pattern is manifest for the selectivity among group Ia cations in that essentially only 11 sequences of cation effectiveness are predicted out of a possible 120 (5!) in each state. These sequences are indicated by the Roman Numerals I to XI above figures 16–18 and, with only minor variations indicated by subscript "a", constitute a quantitative generation of the cationic selectivity rule first proposed elsewhere from an intuitive, qualitative model (Eisenman, et al, 1957c, Rudin and

[32] The important analysis of Cruickshank and Meares (1957) did not consider the effects of a variable $r-$ upon the Free Energy. In fact, these authors concluded that "the free energies of cation exchange between resins and between chloride solutions are of the same order, but show no detailed agreement." However, I have found substantial agreement between the predictions of the above free solution model for a water swollen ion exchanger and the recent careful and extensive data of Reichenberg (1961) who has studied the relationship between ion-exchange selectivity and the number of water molecules per sulphonate group in a series of partially sulphonated polystyrene ion exchangers of widely varying cross-linking. Reichenberg has found experimentally that a unique relationship exists between water per sulphonate and selectivity for the ion studied (K^+—Na^+, K^+—Li^+, and Na^+—Li^+). This relationship predominates over a wide range of deliberately varied cross-linking and specific capacity and invariably indicates an inverse correlation between selectivity magnitude and the number of water molecules per site, which is exactly what would be expected from the above free solution model as discussed below in relation to Fig. 19.

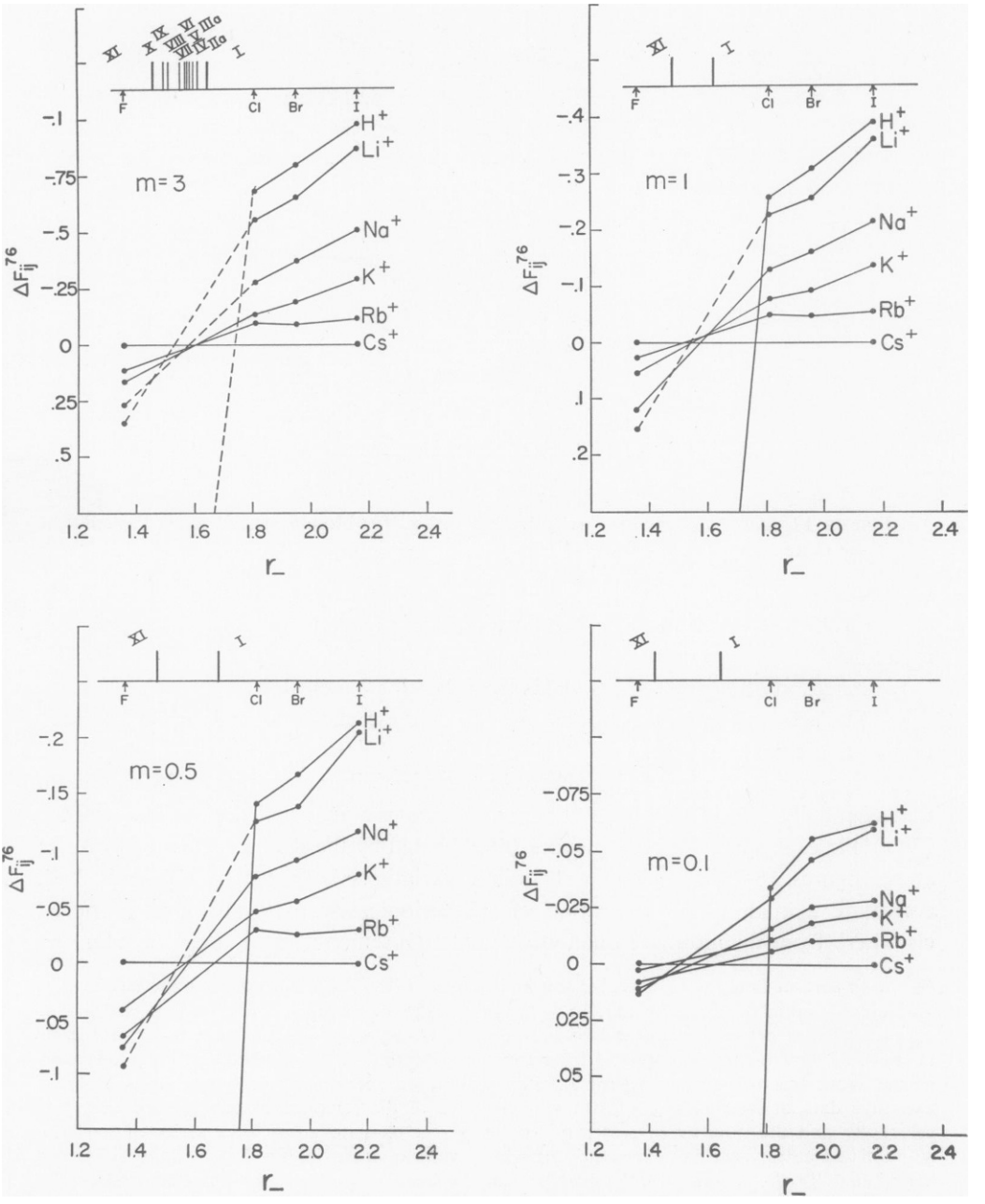

FIGURE 18 $\Delta F_{i,j}°$ selectivities as a function of r_- for water swollen sites at four different molalities, as labelled. ($\Delta F_{i,j}°$ is here labelled "$\Delta F_{i,j}^{76}$" following the terminology of Eisenman (1960a)). The H^+ isotherm takes into account the free energy of association into acid molecules for HF for which significant association occurs. Note the inversions of selectivity between F^- and Cl^-. Dashed lines connect data determined by extrapolation.

Eisenman, 1959. See summaries by Isard, 1959, Mattock, 1961a, pp. 130–134, and Friedman, 1961, Fig. 3). The sequences encountered, as one progresses from right to left in Figures 16–18 are as follows. (The lower the cation on the figure the more it is preferred by the glass, the greater would be its effect on the phase boundary potential, and the higher (under appropriate circumstances) would be its mobility and diffusion coefficient.

I	Cs > Rb > K > Na > Li		
IIa	Cs > K > Rb > Na > Li	or II	Rb > Cs > K > Na > Li
IIIa	K > Cs > Rb > Na > Li	or III	Rb > K > Cs > Na > Li
IV	K > Rb > Cs > Na > Li		
V	K > Rb > Na > Cs > Li		
VI	K > Na > Rb > Cs > Li		
VII	Na > K > Rb > Cs > Li		
VIII	Na > K > Rb > Li > Cs		
IX	Na > K > Li > Rb > Cs		
X	Na > Li > K > Rb > Cs		
XI	Li > Na > K > Rb > Cs		

The above pattern has also been independently confirmed for plausible values of anionic polarizability by Ling (1960a, 1961) in a sophisticated and important quantitative analysis of the interaction energies of a completely particulate linear model of anions, cations, and water molecules (compare his Figure 3 of (1960a) with figures 16–18).

(3) The selectivity for H^+ relative to the group Ia cations is also expected to depend upon r_-, H^+ being markedly preferred at the high field strengths (small r_-) at the left of Figures 16–18, while the alkali cations are preferred at lower field strengths to the right of these figures.[33]

If, instead of considering the effect of variable r_- for a constant "state" of the site, one examines the effect of changing the "state" of sites whose r_- is held constant, the following points are to be noted.

(1) There is no change in pattern of selectivity but merely an effective increase in anionic field strength due to overlap of electrostatic fields of adjacent sites as the sites are brought closer together from the extremely separated situation of Figure 16 to the very tight coordination of Figure 17 (between which limits the actual situation in the glass should fall). Thus, closely packed Br^- type sites have the same

[33] The actual cationic preference relative to H^+ at low field strengths is expected to be less than indicated here, since the species involved at low field strengths is H_3O^+ which comes to lie near Li^+ to the right. This detail was indicated by the dotted line of Figure 7 of Eisenman, 1960a, but is unimportant for the present conclusions. Also an uncertainty in the position of the H^+ isotherm of roughly $\pm$ 0.2 Å should be allowed in view of the crude model used for H^+ in the present treatment.

"effective field strength" as widely spaced F^- type sites as can be seen by comparing Figure 16 with 17. Since variation in the spacing of sites in a glass would be expected to be less important than variation of the individual site field strength, I will not discuss this type of change of "state" of the sites further here.

(2) Of more importance is the effect of the (at present unknown) amount of water-swelling taking place in the vicinity of the glass sites. Fortunately, our analysis indicates that the predominant effect of such water swelling will be on the *magnitude* of selectivity among cations, but will not alter the *pattern* of selection (Eisenman, 1960a; Eisenman, 1961b; Rudin, 1961). This can be seen from the approximately constant pattern of selectivity as a function of r_- at each of the very different conditions of water swelling represented by the molalities of Figure 18. For example, an exchanger having 55.5 water molecules per site would have the selectivities for the various type sites indicated in Figure 18 at $m = 1$. This conclusion is made still clearer by Figure 19 which plots ΔF_{ij}^0 as a function of molality for constant F^-, Cl^-, Br^-, and I^- type sites. The changing molality in Figure 19 of course represents changing the state of the site by altering the number of H_2O molecules in its vicinity and can be seen to produce essentially only an increase of selectivity magnitude as the number of waters decreases (recall the experimental observations of Reichenberg noted in footnote 32.[34]

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[34] It will be noticed by the observant reader that the magnitude of selectivity is very much greater in the water excluding limiting cases of Figures 16 and 17 than at any of the molalities of Figures 18 or 19. Nevertheless, the selectivities among cations can be very respectable in a system having as many H_2O molecules per site as exist in the saturated solution of the alkali halides (i.e. about 5 to 10 H_2O's per site, cf. discussion of Conway (1960), Eisenman (1960b), and Ling (1960b)). This is so since the free energy by which a site in a saturated solution state differs from a site in the anhydrous crystalline state (which latter state the data of Fig. 17 of course represent) is given by equation (20) and can be shown to be small compared to the other energies involved.

$$\Delta F_{ij}^{37\text{sat}} = 2RT(\theta_{JX} - \theta_{IX}) \tag{20}$$

θ_{JX} and θ_{IX} are the osmotic coefficients in the saturated alkali halide solutions and $\Delta F_{ij}^{37\text{sat}}$ corresponds to the free energy change of the reaction

$$\underset{\text{crystal}}{IX} + 55.5/\text{m } H_2O \rightarrow \underset{\text{liquid}}{I^+} + \underset{\text{Saturated solution at molality, } m}{X^-} + 55.5/\text{m } H_2O \tag{21}$$

(State 3) (State 7 sat)

less the free energy change of the corresponding reaction for JX, $\Delta F_{JX}^{37\text{sat}}$.
By combining equation (20) with thermochemical values for the lattice free energies of the alkali halides and their free energies of hydration it is possible to calculate in Figure 20 left the selectivities to be expected for a glass whose sites are assumed to be in a state having the same numbers of water molecules in their vicinity as in the saturated aqueous solution of the corresponding halide. For comparison, Fig. 20 right presents the selectivities for the completely anhydrous state (corresponding to Fig. 17 but presented here using entirely experimental data). Figure 20 right is identical to Fig. 4D of Eisenman, 1960a.

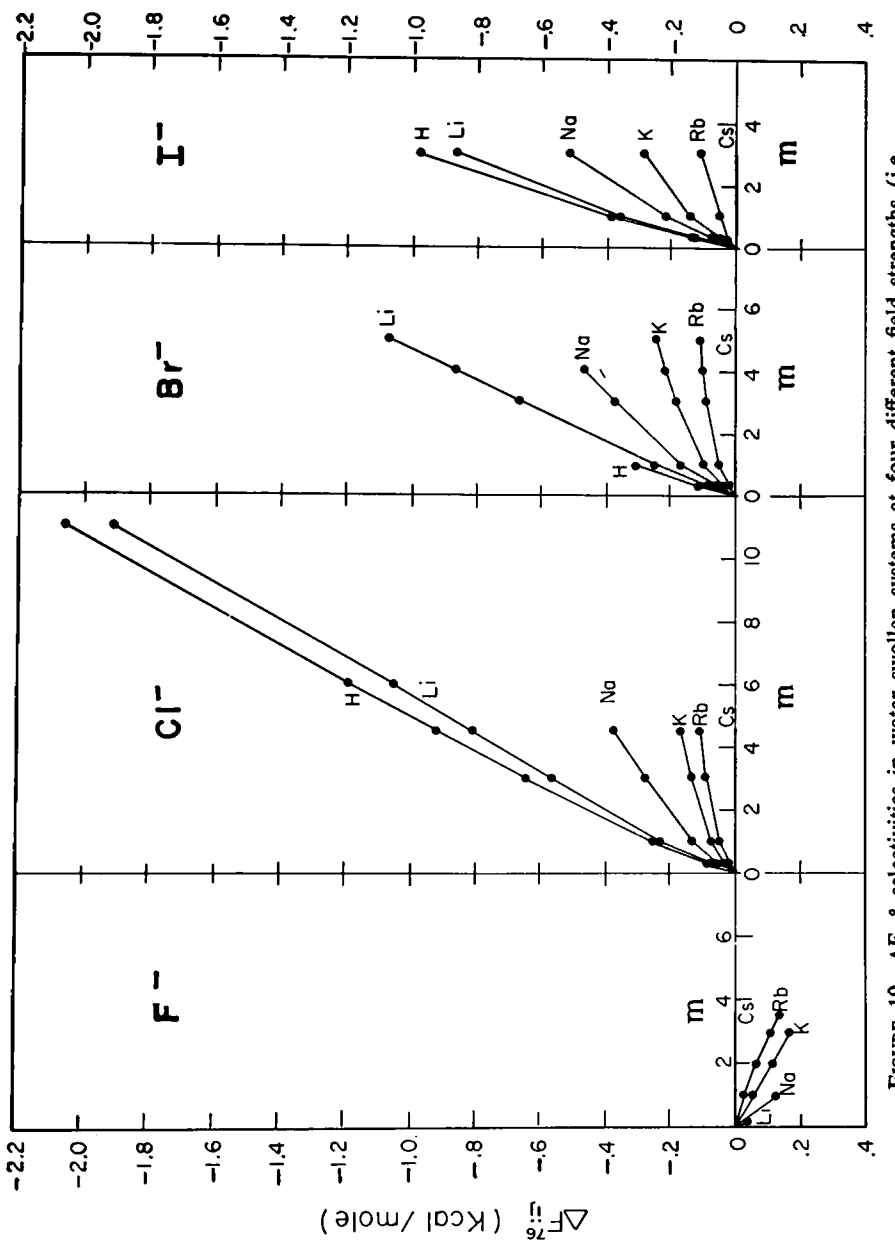

FIGURE 19 $\Delta F_{1,j}^{\circ}$ selectivities in water swollen systems at four different field strengths (i.e. for fluorides, chlorides, bromides, and iodides from left to right) as a function of molality. Described in Text. Note the predominant "spreading" of selectivity with decreasing H_2O's and the lack of tendency to change rank orders.

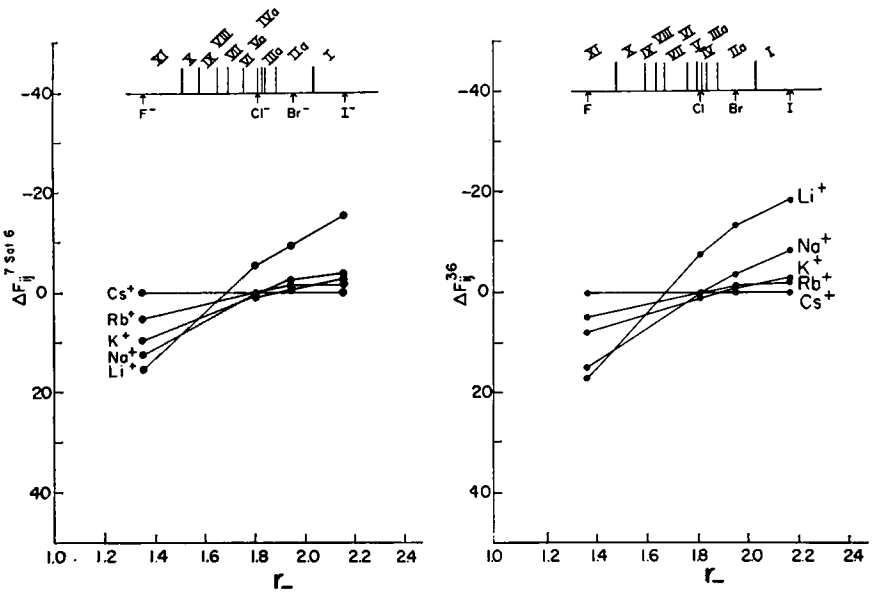

FIGURE 20 *Left.* Free energy selectivities to be expected in a water swollen system having the same molalities as saturated solutions of the various alkali halides. ($\Delta F_{ij}^{7\,sat\,6}$ refers to the free energy change in the terminology of Eisenman (1960a)). *Right,* for comparison are presented the corresponding free energy selectivities for a completely anhydrous system having highly coordinated sites and counterions (i.e. ΔF_{ij}^{36} for the alkali halide crystals as model exchangers). The data of both left and right figures are entirely experimental, although short extrapolations were needed to obtain the osmotic coefficients in the saturated solution. The energies in the left hand figure differ from those of the right by ΔF_{ij}^{87sat} as defined in equation 20.

Application of Theory

From the above considerations it should be clear that H^+ selective glasses on the one hand and alkali cation selective glasses on the other are representatives of a continuous family of generally cation responsive electrodes and differ only as a consequence of the differing field strengths of their sites. (In this regard, recall Figure 9b in which several typical silicate pH glass electrodes were plotted on common coordinates with aluminosilicate alkali cation responsive glasses). The essential requirement for a glass to be alkali cation selective is that it must contain sites of appropriate field strength (i.e., moderate to low) while to be H^+ selective the site field strength must be high. Such a moderate field strength is produced by introducing a triply charged ion into four fold coordination in a glass to yield a site of the type of Figure 13; and we may therefore conclude that we understand why the introduction of Al^{+++} into an alkali silicate glass results in cation responsive glass electrodes. It should now also be clear why B^{+++} and Sc^{+++} can play a role similar to Al^{+++} since they also enter into four fold coordination to produce the $(BOSi)^-$ and $(ScOSi)^-$

analogues of the $(AlOSi)^-$ site.[35] However, $B,^{+++}$ because of its smaller size than Al^{+++} and consequently more intense field, would be expected to produce a lower field strength site than the corresponding alumino-silicate, while the reverse could be expected for Sc^{+++}, thereby decreasing x' and increasing x'' of the footnote on p. 21. On the other hand, since neither Fe^{+++} nor La^{+++} enter into such four fold coordination, their incorporation in an alkali silicate glass does not alter its high field strength characteristics and these ions are without important effects on the selectivity of glass.[36] In the light of the theory, we can also understand why a glass need not contain the cation to which it is primarily responsive, since the only function of this cation is to produce by "screening" the appropriate field strength. Thus sodium silicates and lithium silicates can be markedly H^+ selective, while sodium alumino-silicates can respond preferentially to almost any desired alkali cation, as can the other alkali alumino-silicates (and also silver alumino-silicates (Eisenman, unpublished data)), within the restrictions set by their effective field strength.

It should also now be clear why the cation selectivity of glasses depends upon the ratio of M^+/Al^{+++} and why one can replace the Si^{++++} of a glass by another lattice forming atom such as Ge^{++++} without important effect on electrode properties, since GeO^- sites should have about the same field strength as SiO^- sites. In addition, I think we can now state that we understand why Na^+ to K^+ selectivity is a simple function of M^+/Al^{+++} ratio in the glass (cf. Fig. 10) and why selectivities among the other cations are simple functions of either M^+/Al^{+++} ratio (cf. Figs. 6 and 7) or Na^+ to K^+ selectivity (cf. Figs. 8 and 9). All these observations are direct consequences of the dependence of selectivity upon field strength and of field strength, in turn, upon composition. In consequence, glasses would be expected as a function of either M^+/Al^{+++} or Log $K_{a_{NaK}}$ to follow the pattern of selectivity predicted in Figures 16 to 18; and this expectation is amply fulfilled in the data of Figures 6–9. In this regard, it should be noted that the dependence of selectivity upon Na^+/Al^{+++} ratio, with increasing Na^+/Al^{+++} ratio corresponding to decreasing field strength, is what would be expected if each Na^+ in the glass in excess of Al^{+++} were forced into a lattice breaking SiO^-Na^+ position from which it would "screen" and thereby diminish the negative field strength of the $(AlOSi)^-$ site.[37] Thus, the low field strength which results from high ratio of M^+/Al^{+++} in the glass leads to increased K^+ relative

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[35] Cf. in particular the definitive work of Silver and Bray (1958) on the coordination B^{+++} in Borate and Borosilicate glasses.

[36] On the other hand Lengyel and Csakvari (1960) report that Zr^{4+} produces alkali cation responsiveness when incorporated in a sodium silicate, noting that its coordination number is probably 6. Application of Pauling's electrostatic valence rule (cf. p. 35 above) to the $ZrOSi^=$ site indicates a residual bonding strength of 1/3, or slightly higher than the value of ¼ for the $AlOSi^-$ site. The Alkali Zirconiosilicates should therefore comprise another system of cation responsive glasses, as has been confirmed in preliminary experiments.

[37] Presumably, in contact with aqueous solutions the SiO^-Na^+ would exert its effect as SiO^-H^+ because of ion exchange of Na^+ for the preferred H^+ species.

to Na^+ selectivity and also to numerically low rank orders of selectivity for the other cations.

Also to be expected from this view, is the observation at the right of Figure 10 that, when the ratio of M^+/Al^{+++} falls below 1, selectivity should no longer change importantly since there are then no excess alkali cations available to screen the $(Al\text{–}O\text{–}Si)^-$ sites. It should also be apparent why the same quantitative selectivity rule might be expected to govern lithium alumino-silicates, sodium alumino-silicates, and potassium aluminosilicates (as already noted in regard to Figure 9) since the chief difference among these glasses will be the effective field strength of the $(AlOSi)^-$ site as screened directly by the various cations or indirectly after H^+ exchange (see footnote), as a result of different spacings of SiO^-H^+ groups for each type cation. This also explains why the different cations have a different effectiveness per atom so that 2.5 Li^+ are needed per Al^{+++} to produce the same effective field strength as 1 Na^+, 0.6 K^+, 0.5 Rb^+, or 0.37 Cs^+.

There are, of course, other ways to produce a field strength appropriate for alkali cation selectivity. Thus from considerations of the pK_a of polymerized phosphates it was clear that the phosphate group itself should have such a field strength ($r- \cong 1.4$ Å, cf. Fig. 7 of Eisenman, 1960a). I, therefore, made several phosphate glasses containing only Na, Ca, P, and O (as did P. Mueller at my suggestion). We found that these were indeed cation selective (although they had very high electrical resistance and very poor durability). The behavior of such phosphate glasses, while of little use for practical electrodes, is important as a test of the usefulness of the concept of field strength and the method of obtaining r_- from the pK_a. These observations also indicate clearly that a simple inorganic glass containing only those elements present in the living cell can function as an electrode responsive to the alkali metal cations which supports the assumption that the present analysis is relevant to biology. Similarly, it is important that carboxylic groups can also have a field strength appropriate to alkali cation discrimination (cf. Fig. 7 of Eisenman, 1960a), and it therefore should not be surprising that the air dried collodion films of Wilbrandt (1934) and Ling and Kushnir (1960) function as cation responsive electrodes.

On the other hand, since the selectivity between Na^+ and K^+ is expected to be a monotonic function of r_- in the absence of water-swelling (cf. Figures 16 and 17) and is moreover essentially monotonic at constant water swelling (cf. Fig. 18), it is clear why a measurement of Na^+ to K^+ selectivity alone is useful for characterizing the selectivities among all the other cations as was done in Figure 8 and 9. Since, moreover, one would expect higher Na^+ to K^+ selectivity to correspond usually to higher field strength, it becomes clear why the H^+ to group Ia and the intra-group Ia selectivity patterns of Figures 8 and 9 correspond so well to those predicted by Figures 16–18. However, the observations of Figures 8 and 9 deviate from the predictions of Figures 16–18 in one important way. Namely that while the theory of either completely anhydrous or comparably water-swollen systems requires that the selectivity be a monotonic function of field strength, experiment indicates that in the glass

electrodes there is a maximum in the Na^+ to K^+ selectivity as field strength decreases (cf. data at the right of Fig. 8 and 9 and at the left of Fig. 10). This maximum correlates with a grossly apparent increased imbibition of water in glasses to the right of Figures 8 and 9. Since the free solution model indicates that increasing uptake of water decreases the magnitude of selectivity, such a maximum is exactly what would be expected. The concomitant water-swelling with decreasing field strength appears to set a limit of about 20: 1 on the practical selectivity for K^+ over Na^+ which can be obtained in the present glasses (and also, interestingly, in collodion films as well). Since no such limit exists in the theoretical anhydrous situations of Figure 16 and 17 (which yield extremely high K^+/Na^+ selectivities, indeed), it would seem that in order to develop electrodes of improved K^+ over Na^+ selectivity one should look to means of excluding water from the vicinity of the site, while preserving simultaneously an appropriately low field strength.[38]

Suggestions for Other Types of Ion-Selective Electrodes

Several other consequences of the present theory are worth stating since they suggest beguiling possibilities for the development of specific electrodes for ionic species other than univalent cations. In particular, it should be possible to make *Anion* selective electrodes in at least two ways. On the one hand, if the four-coordinated Al^{3+} could be replaced by another four-coordinated atom which carried a charge of $+5$ instead of $+3$, a *positively* charged site would result. This would be expected to lead to anion-exchange properties and anionic electrode responsiveness. P^{5+} (phosphorous in oxidation state 5) is the prime candidate to replace Al^{3+} in such a situation since its size is proper for four fold coordination. To make such an anion responsive glass one would need to replace Al^{3+} by P^{5+}, while replacing Na^+ by F^- (or OH^-, Cl^-, etc.) In support of the proposition that such a glass might be expected to be anion selective, the reader should note that minerals of the apatite type, $Ca_5F(PO_4)_3$, which have such a site, are known to behave as anion exchangers.

Alternatively, it is conceivable that a complete "mirror image" glass might be makeable in which the various elements are replaced by their oppositely charged counterparts in the periodic table. Thus, the O^{2-} (from period 6) of the structure might be replaced by a cation of $+2$ charge such as Ba^{2+} (from period 2) while the Na^+ (from period 1) would be replaced by F^- (from period 7). Simultaneously the Si^{4+} would be replaced by C^{4-} or by Si^{4-}. It is suggested that such replacements would yield the oppositely charged analogue of the alkali silicate (pH responsive)

[38] A number of possible approaches to this problem are worth suggesting. Incorporation of several mol % of the alkaline earth oxides has proved to be without benefit; but more recently additions of several Mol % ZnO as a result of discussion with M. Nordberg of Corning Glass Works have been encouraging. It is also possible that Scandio-silicates, which should swell less than aluminosilicates (according to Nordberg) might offer a profitable system for study. The replacement of Na^+ in the glass by the comparable sized but much more polarizable and polarizing Ag^+ might lead to a "tightening" of the site structure at low field strength and thereby lead to water-exclusion and higher K^+/Na^+ selectivities.

glasses. Further incorporation of N^{3-} in place of Al^{3+} would yield the oppositely charged analogue of the alkali alumino-silicates.[39]

From the present theory of the origin of ionic selectivity it is also possible to suggest a direction for developing electrodes selective for divalent cations. As mentioned elsewhere in this paper, Garrels and his colleagues (1961) have, in a very important study, found that they can enhance the responsiveness to doubly charged cations of alkali alumino-silicates by introducing large amounts of divalent cations into the glass. Such an addition probably produces an optimal spacing between several sites each bearing a single negative charge and thereby introduces an energetic factor favoring divalent cations relative to monovalent (Eisenman, 1961b). On the other hand, present theory suggests that a system having individual sites each bearing a charge of -2, would exhibit a marked preference for divalent cations compared to the monovalent. Since Be^{2+} and Zn^{2+} are both known to enter into four fold coordination in a number of minerals (cf. Eitel, 1954, page 55), it would be of great interest to make and study simple glasses corresponding to the minerals Zn_2SiO_4 (Willemite) or Be_2SiO_4 (Phenacite).[40] In practice, it would probably be desirable to use slightly more complex glasses than these to which small amounts of alkali cations were also added in order to lower their electrical resistance and also to make the "sites" more accessible for ion exchange by breaking the lattice structure. A prototype for such glasses is the mineral Bertrandite, $Na_2Be_4Si_2O_9$.

4. MISCELLANEOUS PRACTICAL INFORMATION AND BRIEF REVIEW

Survey of Applications

Cation specific glass electrodes are potentially useful wherever a direct measurement of ionic *activity* is desired, whether this be in a simple solution or in a complex ionic mixture *in vitro* or *in situ*. They are also useful for measuring ionic *concentration* provided proper account is taken of the activity coefficient or other standardization procedures are used (e.g. serial dilution and extrapolation to constant amount of cation, or direct empirical calibration). The electrodes offer their most unique advantages, however, when high sensitivity, non-destructive, continuous, or *in situ* recording of ionic activity is desired. Some notion of their possible uses can be obtained by surveying briefly the types of measurements to which they are at present being applied, and this summary will also serve to make available a bibliography to current research.

Cation responsive glass electrodes have been briefly reviewed by Isard (1959), and the high precision measurement of Na^+ and K^+ with glass electrodes is discussed by Friedman (1961), who with his colleagues has pioneered the application of cation

[39] These particular atoms are suggested since they tend to preserve the radius ratios desirable, but not essential, to form stable glasses.

[40] G. Mattock, who first called my attention to the similar coordination of Zn^{++} and Be^{++}, informs me that he is presently studying some glasses having Zn^{++} in 4 fold coordination.

responsive glasses to biological systems. Additional discussion of cation responsive glass electrodes, as well as a general analysis of the factors involved in high precision measurements with glass electrodes will be found in Mattock's forthcoming book (1961, cf. chapter 6).

In addition to the work referred to in the Introduction to the present paper, it should be noted that a number of important papers on cation responsive glass electrodes have also been published by Mattock (1958, 1960, 1961c), Shul'ts and Azo (1955), Shul'ts and Parfenov (1958), Goremykin (1957a, 1957b), Leonard (1959) and Lengyel and Csakvari (1960).

Cation selective glass electrodes have been used to measure Na^+ in simple solutions and in natural waters by Goremykin and Kryukov (1957) and in soil solutions, suspensions, and moist soils by Komarova and Kryukov (1959). They have also been studied in irrigation waters and water extracts of soils by Bower (1959, 1961). Garrels and Thompson have applied Na^+ and K^+ electrodes to characterize the ionic composition of sea-water (1961); and Pommer (1960) has used Na^+ sensitive electrodes to ascertain the titration curve of Montmorillonite. Taulli (1960) applied glass electrodes in determining Na^+ concentration in acidic silica sols, which cannot be analyzed directly by flame photometer. Lepoutre (1961), in experiments preliminary to studies of the alkali cations in liquid NH_3, has found that the glasses function well in alcoholic solutions to temperatures as low as $-50°C$, while Eisenman (1961a) has found them to function adequately as high as $+50°C$.

Cation responsive glasses have also been applied successfully to the study of micelle formation in Na^+ solutions of paraffin chain salts and dyestuffs by Stanley (as quoted by Isard (1959)) and by Epstein and Shedlovsky (1961) and also to the complexing of K^+ with calcium tetraphenylborate by Geyer and Frank (1960). Tomicek and Pulpan (1955) have reported the use of several commercial aluminoborosilicate glasses for the titration of Na^+ and zinc uranyl acetate in alcholic solutions. Garrels, Thompson and Siever (1960) have used Na^+ selective glasses to detect the existence of the $NaCO_3^-$ complex ion and to measure its dissociation constant. Katzman and Gregor (1961) have studied the activities of Na^+ and K^+ salts of acidic lipids in water and in mixed solvent systems.

The glass electrodes have also been applied to detect Na^+ and K^+ binding in protoplasmic extracts by Ungar (1959) and Ungar and Romano (1959) and in the living protoplasm of crab muscle and squid giant axon by Hinke (1959, 1961). Extensive measurements *in situ* of Na^+ and K^+ in circulating blood have been carried out by Friedman and his colleagues (1958, 1959a, 1959b, 1960, 1961; see also Jamieson and Friedman (1961)), while Meyer et al. 1961a, 1961b, (see also Gotoh, et al. 1961) have studied these ions at the surface and within the cerebral cortex and in the circulating blood. Sugioka (1961) has recently used the electrodes to study the Na^+ to K^+ levels in the blood stream over long periods of time. Portnoy, Thomas, and Gurdjian (1961a) in collaboration with W. Angst have developed glass insulated "flow-through" and "dip type" electrodes of a particularly elegant design and desirable

working properties for both Na^+ and K^+. They have also electronically opposed the output of the Na^+ electrode against the K^+ electrode to compensate for fluctuations in Na^+ in Na^+–K^+ mixtures and reduced the error in measuring K^+ by direct recording to 0.25 mEq/liter (Portnoy et al. (1961b)).

Further work is in progress in a number of other laboratories; but the above list should indicate the types of uses the electrodes are finding. I should like, therefore, to bring this paper to a close with a few practical observations and hints regarding the use of the electrodes.

Electrode Insulation and Design

The principle concern in electrode design is in providing adequate insulation for the relatively high resistance, cation responsive glass membrane (10^6 to 10^{10} ohms, depending upon electrode design). The problem of insulating glass electrodes is chiefly one of preventing the potential between the two sides of the glass from being "short circuited" out. To prevent such short circuits it is necessary to guard against not only the more obvious connecting pathways but also surface conducting pathways between the two sides of the glass. Secondarily, it is often desirable to localize the response to a particular region of the electrode (e.g. to the tip of a microelectrode passing through a conducting medium). All methods useful with the usual pH responsive glass electrodes should be applicable to cation responsive glasses whose specific resistivity is of the same order of magnitude and sometimes lower than the usual pH responsive compositions. The usual method of insulating a glass electrode is to use an *external* insulation such as paraffin (Eisenman, et al. 1957a) or a non-conducting glass (cf. Dole (1941), Bates (1954), Kostyuk and Sorokina (1960, 1961), and Mattock (1961)). These methods have been applied with great ingenuity by Friedman, et al (1958, 1960, 1961) to the construction of metal-connected flow-through and dipping electrodes; by Hinke (1959, 1961) to the preparation of glass insulated micro-electrodes; and by Portnoy, et al. (1961a) to particularly elegant and simple glass insulated flow-through and dip electrodes. With the advice of H. P. Hood and M. Nordberg of Corning Glass Works, I have found that Corning type 7510 soft lead glass as well as the slightly harder type 7800 seals easily around tubing of the Na^+ selective NAS 11–18 glass[41] at very low temperatures by the "compression seal" method of Angst as described by Portnoy, et al. (1961a) in a manner entirely analogous to that by which platinum is sealed into glass. On the other hand, I find that the usual K^+ responsive glasses (whether sodium alumino-silicates or potassium alumino-silicates) can be fused directly to Corning type 0120 glass or General Electric (British) type L–I glass; although Hinke (1959, 1961) has reported that a satisfactory seal

[41] NAS 11–18 glass (Eisenman, et al., 1957a) can be obtained in experimental quantities from M. Nordberg, Corning Glass Works, Corning, New York.

could not be made in fabricating micro K^+ selective electrodes with L-I glass.[42]

However, a simpler and possibly more versatile method of insulating electrodes is to insulate them *internally* by layering a non-conducting fluid over the saline-filled region from which one wishes to record, thereby localizing the response to the saline filled region. Contact with the internal saline can then be made with the tip of an otherwise insulated wire or by means of a saline filled glass micro-pipette (see below). I have tested the localization of response to the saline filled tip of such electrodes having outside diameters of about 0.5 mm and found it quite satisfactory. Such an internal insulation should be particularly useful in constructing ultra-micro glass electrodes with a minimum of technical difficulties. For example, to prepare such an electrode one would pull out a finely tapered tube of cation responsive glass and fill it with a non-conducting fluid (e.g. a low viscosity silicone). The tip would then be sealed by heating, leaving the electrode full of non-conducting fluid. To establish electrical connection with the inside of the glass, one could insert into the tip of the electrode a fine saline-filled pipette of any convenient glass and eject a little of the saline so as to fill the tip. The pipette could then be left in place and cemented to the cation responsive glass to form the completed electrode. Since the cation responsive glass of such an electrode is not subjected to the strains of sealing to an external insulating glass, it may be possible to make the wall very thin and thereby achieve electrodes considerably smaller than those of Hinke without an appreciable increase in their resistance. In this way it may be possible to approach the minute size achieved by Kostyuk and Sorokina (1960, 1961) with pH responsive glasses. Since the above construction does not require fusing the cation responsive glass to an insulating glass, it involves a minimum of technical skill and manipulation. Such an internally insulated electrode could be used not only as a microelectrode but might also be inserted directly through the wall of a blood vessel (or into any other tubing through which solutions are flowing) and in this way be used in place of the more complicated "flow-through" type electrodes.

Some Recommended Compositions for Na^+ *and* K^+ *Measurement*

The most highly Na^+ selective glasses relative to K^+ and H^+ occur in the lithium alumino-silicate system, in which electrodes more than 100,000 times as selective for Na^+ as for K^+ exist (cf. Fig. 10). Such highly Na^+ specific glasses, however, are more sluggish in their response to Na^+ than glasses of more modest Na^+ selectivity such as NAS 11-18. Extremely Na^+ specific glasses also appear to manifest a "negative error" in the presence of high concentrations of K^+ (Eisenman, 1961a). For these reasons, NAS 11-18 or a lithium aluminosilicate of comparable Na^+/K^+ selectivity (i.e. of the order of 1,000 to 1 such as Electronic Instruments', Ltd. BH

[42] At the 1961 Gordon Conference on Glass, Carter (1961) described an elegant method of fabricating micro-pH glass electrodes using a silver "stain" as external insulation. Carter believes that this method should also be applicable to alkali cation responsive compositions.

104 type or Beckman Instruments' type 78178) are more useful at present unless one requires unusually high Na^+ specificity.[43]

For K^+ selectivity, satisfactory sodium alumino-silicate glasses have been made from a starting composition of 27 Mol % Na_2O and between 3 and 7 Mol % Al_2O_3. (The greater the Al_2O_3 the more stable the glass but the lower the K^+/Na^+ selectivity). If one prepares such glasses at low temperature in large amounts (as has been done by H. P. Hood and M. Nordberg of the Corning Glass Works), then one can make a quite satisfactory glass starting with an initial batch of 27 Mol % Na_2O as carbonate; 3 Mol % Al_2O_3 as hydroxide; and 70 Mol % SiO_2. Tubing of this glass has been made available at nominal cost by the Corning Glass Works, and hopefully, will continue to be available. The addition (suggested in a discussion with M. Nordberg) of 3 Mol % ZnO to the above glass in place of SiO_2 appears to improve its chemical durability considerably and results in a very satisfactory glass having a 10 fold preference of K^+ over Na^+ (and in an occasional electrode as high as 30 fold). Other useful K^+ glasses (having selectivities on occasion as high as 30 : 1 for K^+/Na^+) exist in the K_2O–Al_2O_3–SiO_2 system. A useful series of such KAS glasses have been made by the Corning Glass Works at the present author's suggestion containing (initial composition) 20 Mol % K_2O and between 3 and 7 Mol % K_2O. These glasses have a somewhat higher electrical resistivity than the corresponding sodium aluminosilicates, but according to Portnoy et al. (1961b) are particularly suitable for fabrication into electrodes by the techniques of Angst. The K_2O–Al_2O_3–SiO_2 glasses are also obtainable from Corning. It should also be noted that the Beckman 78137 electrode (which is a sodium aluminosilicate glass of nominal composition NAS 27–4 (J. Leonard, personal communication), having properties not far from those (cf. NAS 27–4 in Eisenman et al. 1957a, Fig. 3) to be expected from its anticipated analytical chemical composition, is in practice two to three times more selective for K^+ than Na^+. It can therefore be used as a K^+ electrode in many applications.

Response to Other Cations

The above glasses have a much lower sensitivity to doubly charged cations than to the alkali metal cations; but they do respond with the expected slope of $RT/2F$ in pure solutions of the former (Tendeloo, 1942; Eisenman, 1961a). Garrels, Sato, Thompson, and Truesdell (1961), however, have recently made a most important discovery of the existence of a class of glasses having important sensitivity to doubly charged cations such as Ca^{++}. Garrels, Sato, Thompson, and Truesdell have also extended the present author's derivation of equation (1) and developed an electrode equation valid for the potential in 1–2 mixtures.

[43] G. Mattock (1961b) informs me that refinements in the technique of preparing electrodes of the highly Na^+ selective Electronic Instruments', Ltd. BH68 Lithium aluminosilicate glass have greatly improved its speed of response. A detailed characterization of the properties of this glass were presented by Mattock (1961c) at the IMEKO conference in Budapest.

The usual cation responsive electrodes have been found to be insensitive to such anions as Cl^-, OH^-, HCO_3^-, NO_3^-, acetate$^-$, CO_3^{--}, SO_4^{--}, thiosulphate, ferrocyanide, ferricyanide (Eisenman, et al. 1957), Br^-, I^-, (Eisenman, 1961a) or PO_4^{---} (Mattock, 1961). However, Azide may have some effect upon the glass (Bartley, 1960).

As mentioned above, the electrodes also respond to NH_4^+, substituted amines, and amino acids (with some intriguing specific difference among the latter). The response to these cations and cationic groups probably underlies the difficulties which have been observed when using the electrodes in high concentrations of proteins or amino acids (Ling, Kushnir and Horowitz, 1960). On the other hand, in protein concentrations and composition such as exist in plasma there appear to be no difficulties (Eisenman, et al. (1957a); Friedman, et al. (1958, 1959a, 1959b, 1960, 1961; Sugioka (1961)); nor does obvious protein "poisoning" occur in brain extracts (Ungar, 1959) nor when recording intracellularly (Hinke, 1959, 1961).

The electrodes are also highly responsive to Ag^+ and Tl^+, as mentioned previously and discussed elsewhere (Eisenman, 1961a).

Suggestions for Obtaining Maximum Precision

Glass electrodes (and also KCl; calomel reference electrodes) are often slow to recover from thermal, mechanical, and electrical shock. They also sometimes show hysteresis on exposure to extreme acidity or alkalinity. Such phenomena are generally only important if one wishes to measure with greater accuracy than ± 0.5 mV.[44] For highly precise measurements it is necessary to maintain constant temperature of reference as well as glass electrodes and of all test solutions. It is also desirable to use standard solutions having a composition fairly close to that of the unknown and avoid severe changes in concentrations, such as occur when rinsing with distilled water. One should also avoid mechanical shock to the glass and reference electrodes such as may occur upon dipping into and out of solutions. Perhaps the most nearly ideal way in which to accomplish all of these ends is to change solutions by flowing past the electrodes. With such precautions, a number of investigators have achieved accuracies in measuring Na^+ and K^+ activities of the order of $\pm 0.2\%$ (Friedman, et al., (1960), Friedman (1961), Garrels (1961), Portnoy (1961)). The reader should note that this corresponds to a measurement of pH to the third decimal place, no mean accomplishment!

APPENDIX:

THERMODYNAMIC DERIVATION OF EQUATIONS GOVERNING PHASE BOUNDARY POTENTIALS AND ION EXCHANGE EQUILIBRIA

The following treatment is based upon elementary thermodynamics and follows the

[44] In this regard note that ± 0.1 pNa corresponds to ± 5.8 mV or $\pm 26\%$ equivalent concentration change; $\pm .01$ pNa corresponds to ± 0.58 mV or $\pm 2.3\%$ concentration change; and $\pm .001$ pNa corresponds to ± 0.058mV or $\pm 0.2\%$ concentration change.

outline of B. P. Nicolskii's (1937) derivation of the basic solution behavior of the H^+ specific glass electrode.[45] It differs from Nicolskii's treatment only by incorporating in the derivation the assumption (found to be experimentally valid for a wide variety of systems and discussed in detail elsewhere by Eisenman, 1961c) that the activity of a species within a phase is proportional to the n^{th} power of its concentration in the phase. (Nicolskii assumed the 1st power).

Definitions:

$\bar{\mu}_i$ = ionic electrochemical potential of the ion, I^+.
μ_i = chemical potential of the ion, I^+.
ψ = electrostatic potential of the aqueous phase
ψ' = electrostatic potential of the glass
Z_i = ionic valence (= 1 for univalent ions)
F = Faraday constant
R = Gas constant
T = Absolute temperature

The Superscript ^{0} denotes arbitrary standard state. Parentheses denote activities. Primed quantities refer to the glass phase.

General Assumptions:

Reversible equilibrium is reached between the two phases

The same constant number of sites $N^{0'}$ is available to each exchanging cation.

Macroscopic electroneutrality is preserved, which requires that $N_i' + N_j' = N^{0'}$ and that therefore the sum of the mole fractions of $I^{+\prime}$ and $J^{+\prime}$ be unity (as expressed in (35)).

Derivation

Consider the general ion exchange reaction, written previously as equation 8, and here rewritten as equation (22)

(22) $$I^+ \text{ (glass)} + J^+ \text{ (aqueous)} \leftrightarrow J^+ \text{ (glass)} + I^+ \text{ (aqueous)} + \Delta F_{ij}^0.$$

At equilibrium the ionic electrochemical potentials in the glass (primed) and in water (unprimed) must be the same, hence,

(23) $$\bar{\mu}_i = \bar{\mu}_i'$$

(24) $$\bar{\mu}_j = \bar{\mu}_j'.$$

[45] A summary of Nicolskii's derivation is found in Dole (1941) on pages 272–274. I am indebted to G. Karreman for discussion of the present derivation, as well as for his collaboration in extending the present treatment to a more general form to be presented elsewhere (Eisenman, 1961c).

But the ionic electrochemical potential $\bar{\mu}_i$ is defined in terms of the chemical potential, μ_i and the electrostatic potential ψ as

$$\bar{\mu}_i = \mu_i + z_i F\psi. \tag{25}$$

We obtain for $Z_i = 1$, since the electrostatic potential difference $(\psi' - \psi)$ is the same for all ions,

$$F(\psi' - \psi) = \mu_i - \mu_i' = \mu_j - \mu_j' \tag{26}$$

where $(\psi' - \psi)$, the difference of potential between glass and water, will henceforth be written as E.

Now the chemical potential μ_i is related to the activity (I^+) and the chemical potential μ_i^0 in an arbitrary standard state by

$$\mu_i = \mu_i^0 + RT \ln (I^+) \tag{27}$$

$$\mu_j = \mu_j^0 + RT \ln (J^+). \tag{28}$$

Incorporating (27) in (26) we have an equation for the phase boundary potential, E,

$$FE = \mu_i^0 - \mu_i^{0\prime} + RT \ln \frac{(I^+)}{(I^{+\prime})} \tag{29}$$

where $(I^{+\prime})$ signifies the activity of I^+ in the glass while (I^+) represents the activity in water.

To this point we have followed Nicolskii completely. If we now assume that the activities of the ions in the glass are proportional to the n^{th} power of their mole fractions, as indicated in (30) and (31)

$$(I^{+\prime}) = X_i'^n \tag{30}$$

$$(J^{+\prime}) = X_j'^n, \tag{31}$$

we obtain for the potential

$$FE = \mu_i^0 - \mu_i^{0\prime} + RT \ln \frac{(I^+)}{X_i'^n} \tag{32}$$

On the other hand, inserting (30) and (31) in the definition of the thermodynamic equilibrium constant obtainable from combining equations (27) and (28) with (26),

$$RT \ln K = \mu_j^0 - \mu_i^0 + \mu_i^{0\prime} - \mu_j^{0\prime} = RT \ln \frac{(I^+)(J^{+\prime})}{(J^+)(I^{+\prime})} \tag{33}$$

yields the general equation 34 for the ion exchange equilibrium as reported briefly previously (Eisenman, et al. (1957); and discussed elsewhere (Eisenman, 1961c).

$$RT \ln K_n = RT \ln \frac{(I^+)}{(J^+)} \left[\frac{X_j'}{X_i'}\right]^n \tag{34}$$

An equation of the same essential form as equation 34 was discovered by Rothmund and Kornfeld (1917) to describe empirically the ion-exchange behavior of alumino-

silicate permutites. I have found equation 34 to describe satisfactorily many other ion-exchange data (as will be discussed elsewhere as will also the theoretical significance of such equations (Eisenman, 1961c)). For the special case in which $n = 1$, equation 34 describes an "ideal" equilibrium in which the equilibrium constant is independent of the fraction exchanged.

The condition of macroscopic electroneutrality of the glass phase,

$$X_i' + X_j' = 1, \tag{35}$$

may be inserted in (34) to yield

$$RT \ln K_n = RT \ln \frac{(I^+)}{(J^+)} \left[\frac{1 - X_i'}{X_i'}\right]^n, \tag{36}$$

which may be rearranged and simplified to give

$$\frac{(I^+)}{X_i'^n} = [(I^+)^{1/n} + K_n^{1/n}(J^+)^{1/n}]^n. \tag{37}$$

When (37) is inserted into equation (32) for the potential, one obtains

$$FE = \mu_i^0 - \mu_i^{0\prime} + RT \ln [(I^+)^{1/n} + K_n^{1/n}(J^+)^{1/n}]^n \tag{38}$$

or, dividing by the Faraday,

$$E = E^0 + \frac{RT}{F} \ln [(I^+)^{1/n} + K_n^{1/n}(J^+)^{1/n}]^n, \tag{39}$$

which will be recognized as being identical *in form* to equations (1) and (11) of the present paper. For the special case in which $n = 1$, equation 39 reduces to Nicolskii's equation.[46] This represents the fact that in Nicolskii's derivation he assumed the activities of the ions in the glass to be proportional to the first power of their adsorbed amounts, while the n^{th} power has been assumed here.

Equation (37) can be rewritten in the form of a general adsorption isotherm as

$$X_i'^n = \frac{(I^+)}{[(I^+)^{1/n} + K_n^{1/n}(J^+)^{1/n}]^n} \tag{40}$$

or, taking the n^{th} root and rearranging slightly, as

$$X_i' = \frac{1}{1 + \left[K_n \frac{(J^+)}{(I^+)}\right]^{1/n}} \tag{41}$$

or, dividing top and bottom by

$$\left[K_n \frac{(J^+)}{(I^+)}\right]^{1/n},$$

[46] In this form it is also identical to Dole's (1941) simplest equation and also to the form of the Henderson (or Planck) diffusion potential equation as derived by Michaelis (1925) Marshall and Krinbill (1952) and Dray and Sollner (1956).

as

$$X_i' = \frac{\left[\frac{1}{K_n}\frac{(I^+)}{(J^+)}\right]^{1/n}}{\left[\frac{1}{K_n}\frac{(I^+)}{(J^+)}\right]^{1/n} + 1}. \tag{42}$$

For the special case in which $n = 1$ equation (41) is identical to the ion exchange isotherm derived by Boyd, Schubert, and Adamson (1947) following Langmuir's isotherm for a binary gas mixture. The Boyd-Langmuir constants b_1, b_2 and K are related to those of equation (41) by

$$\frac{b_2}{b_1} = K_n; \qquad K = 1. \tag{43}$$

At constant (J^+) when $n = 1$ equation 41 can be shown to reduce to the usual Langmuir form as given by Glasstone (1946) p. 1199,

$$\frac{x}{m} = \frac{k_1 k_2 P}{1 + k_1 P} \tag{44}$$

More generally, for all n at constant (J^+), in the special case limit of a small fraction of the exchanger in the I^+ form, equation (42) gives rise to the Freundlich isotherm

$$\frac{x}{m} = kP^{1/n} \tag{45}$$

as written by Glasstone (1946) p. 1196. For in this limit, equation (42) may be reduced to

$$X_i \underset{\substack{\lim X_i \to 0 \\ (J^+ \text{ constant})}}{} = \left[\frac{1}{K_n(J^+)}\right]^{1/n} (I^+)^{1/n} \tag{46}$$

Wherein the k of Freundlich's isotherm is given by the constant

$$\left[\frac{1}{K_n(J^+)}\right]^{1/n},$$

and the activity of (I^+) in solution corresponds to the partial pressure, p, of equation 45.

In cooperation with J. U. Casby and G. Karreman, it has also been possible to prove that K_n as extracted from any data fitting equations 10 or 34 is identical to the thermodynamic equilibrium constant, K_T, which would otherwise be obtainable only by laborous graphical integration (see for example Argersinger, Davidson and Bonner (1950)). As demonstrated by Argersinger, Davidson, and Bonner, the Gibbs-Duhem relationship yields an equation for K_T in terms of the apparent equilibrium constant, Ka, at various mole fractions of exchange, X',

$$\ln K_T = \int_0^1 \ln K_a \, dX'. \tag{47}$$

But Ka, which is defined as

$$K_a = \frac{(I^+)}{(J^+)} \frac{X_i'}{X_i'} \tag{48}$$

can be written in terms of K_n, n, and X_i' by eliminating $(I^+)/(J^+)$ of equations (48) and (34) to yield

$$K_a = \left(\frac{1 - X_i'}{X_i'}\right)^{n-1} K_n \tag{49}$$

which can be substituted in (47) to yield

$$\ln K_T = \int_0^1 \ln K_n \left(\frac{1 - X_i'}{X_i'}\right)^{n-1} dX_i' \tag{50}$$

Equation (50) can then be simplified as follows

$$\ln K_T = \int_0^1 \ln K_n \, dX_i' + \int_0^1 (n - 1) \ln (1 - X_i') \, dX_i' - \int_0^1 (n - 1) \ln X_i' \, dX_i' \tag{51}$$

which reduces to

$$\ln K_T = X_i' \ln K_n]_0^1 = \ln K_n. \tag{52}$$

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